Quadrature $\Sigma\Delta$ Modulators with a Dynamic Element Matching Scheme

Roberto Maurino, Member, IEEE, and Christos Papavassiliou, Member, IEEE

Abstract—This brief presents a new topology of a multibit quadrature band-pass sigma-delta modulator which employs a simple dynamic element matching (DEM) technique in order to reduce the effects of path mismatch, namely aliasing in the signal band of the mirror images of the signal and of the quantization noise. The DEM scheme results in a reduction of the aliasing of the quantization noise mirror image while it reduces the input signal mirror image alias problem to a self-image problem. It is shown that the self-image can be completely removed in switched-capacitor (SC) implementations by using the same capacitors to sample the input and the reference of the feedback DACs. Moreover, a simple method for extending low-pass mismatch noise shaping techniques to the complex band-pass case is proposed for the case of multibit feedback DACs.

Index Terms—A/D converters, band-pass sigma-delta modulation, I/Q signal processing, dynamic element matching.

I. INTRODUCTION

Quadrature $\Sigma\Delta$ (Q$\Sigma\Delta$) modulators have been shown to be advantageous over real band-pass $\Sigma\Delta$ modulators [1], [2]. However, Q$\Sigma\Delta$ modulators suffer from path mismatch that causes both interferers and quantization noise in the mirror image band to alias into the desired signal band, thus degrading the SNDR of the converter [2]. In the past, an additional complex integrator has been introduced to place a zero in the noise transfer function (NTF) at the image band, in order to attenuate the image quantization noise [2]. This complicates the design of the NTF and it can actually hinder the noise shaping in the desired band [1]. For the signal image, digital adaptive techniques have been proposed [3] and recently a dynamic matching technique has been reported [4]. However, digital techniques require additional hardware, whereas the dynamic matching technique of [4] has the downside of raising the noise floor of the converter.

In the following, a new Q$\Sigma\Delta$ modulator topology with a simple dynamic element matching (DEM) scheme is introduced, which has two major benefits. First, it reduces the amount of the quantization noise mirror image aliased in the signal bandwidth. Second, it removes the alias of the mirror signal, but it generally introduces a self-image component. However, self-image attenuation figures of more than 25 dB are easy to achieve and normally acceptable [5]. Moreover, a particular SC topology that suppresses the self-image component is identified. In section II, the effect of path mismatch in a complex system is briefly analyzed. Then, section III discusses how path mismatch can be tackled in a Q$\Sigma\Delta$ modulator with a simple DEM scheme. An extension of the technique to multibit converters is illustrated. Finally, in section IV the performance of both a 2nd and 3rd order multibit Q$\Sigma\Delta$ modulator employing the proposed DEM scheme is investigated with behavioral simulations.

II. MISMATCH IN COMPLEX SYSTEMS

Let’s consider first a unity gain block with mismatch coefficient $\delta_r$ in the real path and mismatch coefficient $\delta_i$ in the imaginary path (Fig. 1(a)). We can write:

\[
\begin{align*}
y_r(n) &= (1 + \delta_r)x_r(n) \\
y_i(n) &= (1 + \delta_i)x_i(n)
\end{align*}
\]

where $n$ is the clock cycle index, which is introduced because the DEM scheme described hereafter applies only to discrete time systems, such as SC or switched-current (SI) circuits.

The mismatch coefficients $\delta_r$ and $\delta_i$ can be interpreted as the sum of a “common mode” component $\delta_{CM}$ and a “differential mode” component $\delta_{DIFF}$, as follows:

\[
\begin{align*}
\delta_r &= \delta_{CM} + \delta_{DIFF} \\
\delta_i &= \delta_{CM} - \delta_{DIFF}
\end{align*}
\]

It follows that (1) can be rewritten as:

\[
y(n) = (1 + \delta_{CM})x(n) + \delta_{DIFF} x^*(n)
\]

where $x(n) = x_r(n) + jx_i(n)$, $y(n) = y_r(n) + jy_i(n)$ and $* \text{ indicates conjugation}$, namely $x^*(n) = x_r(n) - jx_i(n)$.

According to (3), the common mode component affects only the gain value; single loop $\Sigma\Delta$ modulators are known to be robust against coefficient gain errors. On the other hand, the differential component introduces an error proportional to...
the conjugate of the input signal. The latter will be shown to severely degrade the performance of QΣΔ modulators.

Let’s now introduce the swapper. A swapper operates as follows: when the clock cycle index \( n \) is odd, the real part of its output is the imaginary part of its input, and the imaginary part of its output is the real part of its input; when the clock cycle is even, the swapper is just a straight connection, namely the output real part is equal to the input real part and the output imaginary part is equal to the imaginary input part.

When we insert the gain stage of Fig. 1(a) between two swappers we obtain the system of Fig. 1(b), for which we have:

\[
y(n) = (1 + \Delta_{cm})x(n) + (-1)^n \Delta_{DF} x^*(n).
\] (4)

Equation (4) is identical to (3) but now the conjugate of \( x \) has been multiplied by the sequence \((-1)^n\), that is, it is mixed with a cosine at \( f_s/2 \), where \( f_s \) is the sampling frequency.

This feature can be usefully exploited to counteract the effect of the differential mismatch in a particular class of QΣΔ A/D converters, as shown in the following section. In the rest of this brief, only differential mismatch will be considered.

### III. Complex ΣΔ A/D Converter with DEM Scheme

We consider modulators with the topology illustrated in Fig. 2 [6],[7]. The quantizer has been modelled, as usual, as a unity gain which adds the quantization noise \( q \). Notice that when \( C_0 \) is equal to 1 the input of the loop filter is just shaped quantization noise, without any signal component. All the signals in Fig. 2 are complex and the integrators \( I \) have transfer function \( j/(z - j) \).

#### A. Mismatch in the Loop Filter

In order to investigate the effect of mismatch in the loop filter, we follow a small perturbation approach, ignoring second order effects. Namely we work out the signal and the quantization noise at the various points of the circuits in the ideal case, that is without mismatch, and then for each gain block we use (3) or (4) to produce an estimate of the disturbance introduced by the mismatch. Fig. 3(a) shows a possible implementation for the complex integrators used in Fig. 2. \( G_r \), \( G_i \), and \( G_u \) are unity gain blocks with differential mismatch coefficients \( \delta_r \), \( \delta_i \), and \( \delta_u \), respectively. Then, according to (3) or (4) we can model the integrator of Fig. 3(a) as an integrator without mismatch and an error \( e \) added to its input, as shown in Fig. 3(b). If the integrator of Fig. 3(a) uses simple gain stages of the type of Fig. 1(a), the error is given by:

\[
e(n) = (\delta_r + \delta_u)x(n) + (\delta_i + \delta_u)y^*(n)
\] (5)

whereas with swappers (Fig. 1(b)), it is given by:

\[
e(n) = (-1)^r(\delta_r + \delta_u)x(n) + (-1)^i(\delta_i + \delta_u)y^*(n)
\] (6)

Let’s consider a 2\(^{nd}\) order modulator of the type of Fig. 2 with \( C_0, C_1, \) and \( C_2 \) set to 1,2, and 1, respectively. Notice that the contribution to the output of the modulator from an error injected at the input of the first integrator is obtained as:

\[
Y_0(z)|_{E_i} = z^{-1}(2j + z^{-1})E_1(z).
\] (7)

The transfer function of (7) has a modulus of 1 at \( f_s/4 \). Thus, in the band of interest, any disturbance introduced at the input of the first integrator appears unattenuated at the output of the modulator.

For an ideal modulator, the input and the output of the first integrator are given by:

\[
X_i(z) = -(1 - jz^{-1})^2 Q(z)
\]
\[
Y_i(z) = -jz^{-1}(1 - jz^{-1})Q(z)^{2}
\] (8)

These are both shaped noise with a notch at \( f_s/4 \). Their spectrum is qualitatively depicted in Fig. 4(a). Recollecting that \( X(f) \) is the Fourier transform of a signal \( x \), then \( X^*(-f) \) is the Fourier transform of its conjugate \( x^* \), it follows that conjugation simply mirrors the power spectrum of a signal around DC (Fig. 4(b)). Thus the conjugates of \( X_i(n) \) and \( y_i(n) \) have strong frequency components around \( f_s/4 \).

This means that with simple gain stages with mismatch, the error introduced at the input of the first integrator is:

\[
E_1(z) = (\delta_{r1} + \delta_{u1})X_i^* (z^{-1}) + (\delta_{i1} + \delta_{u1})Y_i^* (z^{-1}) = \left[ (\delta_{r1} + \delta_{u1})^2 + (\delta_{i1} + \delta_{u1})^2 \right] Q^* (z^{-1})
\] (9)

which can be approximated at \( f_s/4 \) with:

\[
E_1(z) \simeq (-4\delta_{r1} - 2\delta_{u1} + 2\delta_{i1})Q^* (z^{-1})
\] (10)

Since \( q^* (n) \) has also a white spectrum, mismatch tends to fill in the notch with white quantization noise attenuated by a factor proportional to the mismatch coefficients. This is clearly apparent in simulation: Fig. 5 shows the output spectrum (dashdot line) of the 2\(^{nd}\) order modulator where all the mismatch coefficients of the integrators have been set to 1\%, while everything else is kept ideal. On the other hand, when using the gain with swappers (Fig. 1(b)), the mixing of the conjugate of \( x_i(n) \) and \( y_i(n) \) with a cosine at \( f_s/2 \) produces again shaped noise with a notch at \( f_s/4 \) (Fig. 4(c)).

This leads to a considerable improvement, as shown in Fig. 5 (continuous line).

The reason why this scheme does not result in an infinite
notch at \( f_s / 4 \) is that the output of the last integrator in the loop filter of the modulator of Fig. 2 is always unshaped quantization noise; for instance, for the 2nd order modulator:

\[
y_1(n) = q(n - 2)
\]  
(11)

The mismatch coefficients \( \delta_{y_2} \) and \( \delta_{u_2} \) introduce a disturbance at the input of the second integrator given by:

\[
e_2(n) = (-1)^n (\delta_{u_2} + \delta_{y_2}) q * (n - 2).
\]  
(12)

The disturbance \( e_2(n) \) generates a modulator’s output component whose \( z \)-transform is given by:

\[
Y_2(z)_{p_2} = jz^{-1}(1 - jz^{-1})E_2(z).
\]  
(13)

This component is then subtracted at the input summing junction of the modulator and integrated by the first complex integrator, resulting in the following error component at the output of the first integrator:

\[
Y_1(z)_{p_2} = z^{-2}E_2(z)
\]  
(14)

which shows that the error signal \( e_2(n) \) appears at the output of the first integrator simply delayed by two clock cycles. Thus the output of the first integrator is not just first order shaped quantization noise, as stated in (8). When this is taken into account, it is found that the error introduced at the input of the first integrator has a component which is unshaped quantization noise and is given by:

\[
e_1(n) = (\delta_{u_1} + \delta_{y_1})(\delta_{y_2} + \delta_{u_2})q(n - 4).
\]  
(15)

At \( f_s / 4 \), this is the dominant noise contribution due to mismatch in the loop filter. Equation (15) correlates well with simulation results and it has been used in Fig. 5 for the predicted output spectrum (dashed line). Extending (15) to the case of a modulator of order \( N \) of the type of Fig. 2, it is found that the dominant residual term is given by \( q(n) \) for \( N \) even or \( (-1)^n q * (n) \) for \( N \) odd, delayed and with an attenuation \( \alpha_N \) given by:

\[
\alpha_N = \prod_{i=1}^{N} (\delta_{y_i} + \delta_{u_i})
\]  
(16)

Notice that \( q(n) \) and \( (-1)^n q * (n) \) have the same flat power density spectrum. Equation (16) shows that the impact of mismatch is reduced as the order of the modulator increases.

Mismatch coefficients of less than 1% or better are routinely achieved in SC filters. Even for a 2nd order modulator, (16) predicts an attenuation of 68dB or better. Oversampling and, possibly, the use of a multibit quantizer are often sufficient to further reduce the in-band power of \( e_1(n) \) to negligible levels.

B. Mismatch in the Input Signal Branches

While in the modulator topology of Fig. 2 the loop filter deals only with quantization noise, the input branches connecting the input signal \( x \) to the two summing junctions operate only on the input signal itself. Any error introduced in the summing junction in front of the quantizer is attenuated by the gain of the loop filter, and can therefore be neglected. On the other hand, any error introduced at the input summing junction appears unattenuated at the output of the modulator. It is straightforward to show that with a complex input signal \( Ae^{j2\pi f_s / 4 + M}b \), a disturbance signal \( \delta_{in} Ae^{j2\pi f_s / 4 - M}b \) is introduced when applying the DEM scheme of Fig. 1(b), where \( \delta_{in} \) is the mismatch coefficient of the input branch.

This is a self-image of the signal attenuated by \( \delta_{in} \).

C. Mismatch in the Feedback DACs

The feedback DACs process both signal and quantization noise. If the DACs are single bit, then the DEM scheme can be applied. Considering first the quantization noise, since the DACs process only shaped quantization noise, the DEM scheme is very effective and it renders the mismatch of the DACs harmless. On the other hand, the gain mismatch coefficient of the DACs \( \delta_{DAC} \) does contribute to the signal self-image component. Taking into account both the mismatch coefficient of the input branch \( \delta_{in} \) and the mismatch coefficient of the DACs \( \delta_{DAC} \), the attenuation \( \alpha_S \) of the signal self-image is given by:

\[
\alpha_S = \delta_{in} - \delta_{DAC}.
\]  
(17)

For many applications, an attenuation of 25 dB is sufficient and this is easily achieved [5]. Furthermore, (18) suggests that the self-image component can be completely eliminated when \( \delta_{in} \) is equal to \( \delta_{DAC} \). This is possible in SC implementation, by using the same capacitors for sampling the input signal and the DAC reference voltage. This is also advantageous in term of \( kT / C \) noise [8].

Though the topology of Fig. 2 can be implemented with single bit quantizers, it is particularly desirable to use multibit quantizers in order to reduce the dynamic range requirements of the loop filter. Unfortunately, a multibit quantizer poses stringent linearity specifications on the feedback DACs. For low-pass (LP) \( \Sigma \Delta \) modulators, this is normally addressed with mismatch noise shaping techniques [9]. In practice, a noise shaping scheme is implemented by adding an Element Selection Logic (ESL) block in front of the DAC. In LP \( \Sigma \Delta \) modulators, the ESL selects the DAC elements in such a way that the energy of the mismatch noise is moved to high
frequencies. Any of the known LP mismatch noise shaping techniques can be used by previously downconverting the input signal from $f_s/4$ to baseband before feeding the ESL, and by consequently upconverting back to $f_s/4$ the output of the DACs (see Fig. 6(a)). Downconversion is implemented by multiplication with $e^{-jx}$, and upconversion by multiplication with $e^{jx}$, which requires quadrature mixers. By modelling the combination of the ESL logic blocks and the DACs as gain blocks with gain $G$ and mismatch coefficient $\delta_{DAC}$ which add LP shaped mismatch noises $e_{LP}(n)$ and $e_{LP'}(n)$ as in Fig. 6(b), the output is obtained as:

$$y(n) = Gx(n) + G\delta_{DAC} (-1)^n x * (n) + (e_{LP}(n) + je_{LP'}(n))e^{j\frac{\pi}{2}}.$$  \hspace{1cm} (18)

The last term of (13) is LP shaped mismatch noise frequency translated to $f_s/4$, which effectively results in complex mismatch noise shaping at $f_s/4$. The first terms are identical to the output produced by a gain stage $G$ and mismatch coefficient $\delta_{DAC}$ with the DEM scheme applied (compare with (4)).

Notice that also for the multibit case, it is still possible to eliminate the self-image component by using the same unit capacitors that implement the feedback DACs to sample the input signal. This has been verified with circuit level simulations.

IV. SIMULATION RESULTS

100 Monte Carlo behavioural simulations have been run for a 2nd order Q\&\Sigma modulator with a 3 bit quantizer (Fig. 2). All the mismatch coefficients were normally distributed with a standard deviation of 1%. It was assumed that capacitors were shared between the input and the reference, and therefore $\delta_m$ was equal to $\delta_{DAC}$. Fig. 7 shows the average and the worst case in band noise power versus the OSR, for a modulator with ideal feedback DACs with or without the DEM scheme. The DEM scheme results in an improvement of about 40dB. The predicted average power $P_N$ of the residual mismatch noise is found to be equal to $\frac{\Delta^2}{6} \left(\frac{2\sigma^2}{OSR}\right)^N$ by integrating over the signal bandwidth the expected value of the square of (15) (or equivalent for higher order modulators). $\Delta$ is the quantizer step, $\sigma$ is the standard deviation of the mismatch coefficients, which are assumed statistically independent, and $N$ is the modulator order. As the OSR increases $P_N$ becomes the dominant term.

Fig. 7 also shows the results for a 2nd order modulator with mismatch in the elements of the DACs and in the gain blocks. All the mismatch errors were normally distributed with a standard deviation of 1%. This modulator employs the DEM scheme and the complex noise shaping method of Fig. 6, where the ESL blocks implement the data weighted averaging (DWA) algorithm [10], resulting in negligible degradation compared to case of ideal DACs. The largest residual self-image, mainly due to the mismatch of the input feed-forward branch, was found to be 94dB below the input signal.

Finally Fig.8 shows the corresponding results for a 4 bit quantizer 3rd order Q\&\Sigma modulator for the same conditions. The DEM scheme can provide up to 80 dB improvement. However, when mismatch in the DACs is also included, it is apparent that the first order shaping of the DWA is not adequate and a higher order mismatch shaping algorithm would be preferred.

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REFERENCES

Figure 1: (a) Complex unity gain with mismatch. (b) Complex unity gain with swappers.
Figure 2: $N$-th order QΣΔ modulator topology.
Figure 3: (a) Complex integrator block diagram. (b) Equivalent model of complex integrator with mismatch.
Figure 4: Qualitative illustration of DEM scheme for quantization noise (y axis arbitrary units):
(a) Spectrum of $x_1 (\gamma_1)$. (b) Spectrum of the conjugate of $x_1 (\gamma_1)$. (c) Spectrum of mixing product of the conjugate of $x_1 (\gamma_1)$ with a real cosine at $f_S/4$. 
Figure 5: Output spectrum (10 averages) of a 2nd order ΣΔ modulator: ideal, with non-ideal integrators with/without DEM and predicted.
Figure 6: (a) Complex mismatch noise shaped DAC. (b) Equivalent block diagram.
Figure 7: Noise relative to a mid-scale input signal versus 1/OSR for a 2nd order QEA modulator with $C_0$, $C_1$, and $C_2$ nominally set to 1, 2, and 1, respectively: ideal, with and without DEM, and 1% standard deviation for the gain coefficient mismatch, and predicted. Also shown the case of a modulator with DEM and ESL with 1% standard deviation for the unity DAC elements and the unity gain blocks.
Figure 8: Noise relative to a mid-scale input signal versus 1/OSR for a 3rd order QΣΔ modulator with $C_0, C_1, C_2$ and $C_3$ nominally set to 1, 3, 3, and 1, respectively: ideal, with and without DEM and 1% standard deviation for the gain coefficient mismatch, and predicted. Also shown is the case of a modulator with DEM and ESL with 1% standard deviation for the unity DAC elements and the unity gain blocks.