Feedback amplifiers

Motivation:

- Op-amps, transistors, valves are not ideal they have:
  - *Finite port impedances*
  - *Finite gain*
  - *Finite Bandwidth, i.e. gain depends on frequency!*

Our Approach:

- Develop analytical tools to handle feedback connections

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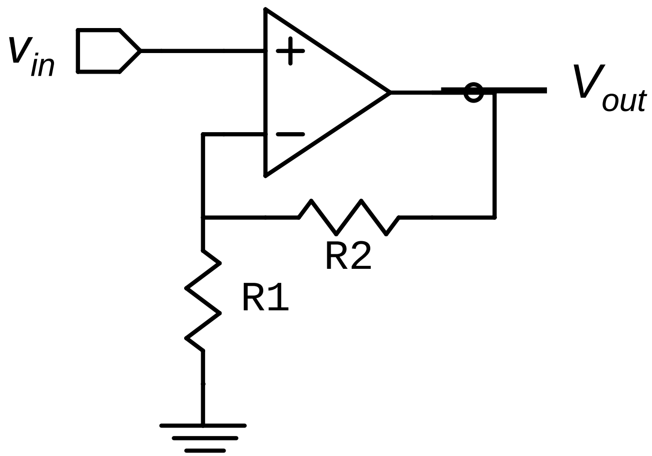
NB: Some figures from Sedra/Smith: Microelectronic Circuits, 4th ed. (Oxford)

Circuits using ideal op-amps

An ideal op-amp is a voltage amplifier with
- infinite voltage gain
- zero input admittance
- zero output impedance
- Zero reverse gain



This definition results in the "golden rules":
- $v_+ = v_-$
- $i_+ = i_- = 0$

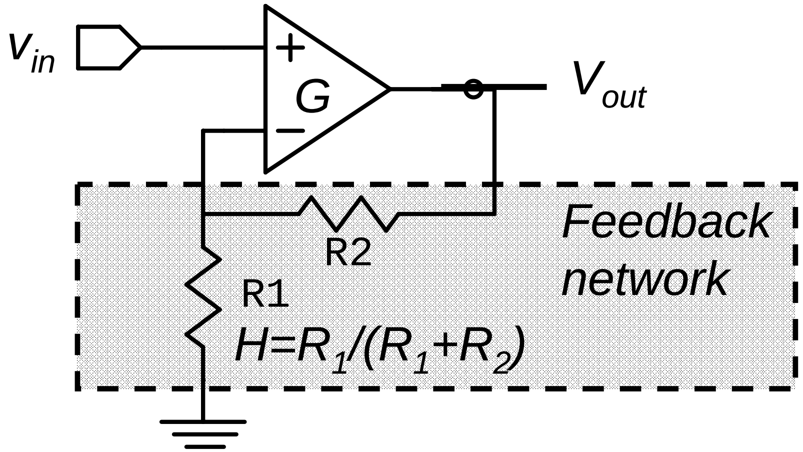
The golden rules usually make the solution of a circuit which uses ideal op-amps very easy.
For example, for the non-inverting amplifier:

$$v_- = v_+ \Rightarrow v_- = v_{in} \Rightarrow v_{out}\left(R_1 / R_1 + R_2\right) = v_{in} \Rightarrow v_{out} = v_{in}\left(1 + R_2 / R_1\right) \Rightarrow$$

$$A_V = 1 + \frac{R_2}{R_1}$$

What if the op-amp has a finite gain?

Circuits using not-so-ideal op-amps



- *Keep all the properties of the ideal op-amp except that the op-amp now has a finite gain G. (G may be complex, or a function of frequency!)*
- *The network connecting the output and the input is an ideal voltage divider (since both Y_{in}=0 and Z_{in}=0) with gain $H=R_1/(R_1+R_2)$ from output to input.*
- *The amplifier must amplify its input by G: $V_{out}=G(V_+-V_-)$*
- *We can now solve the circuit :*

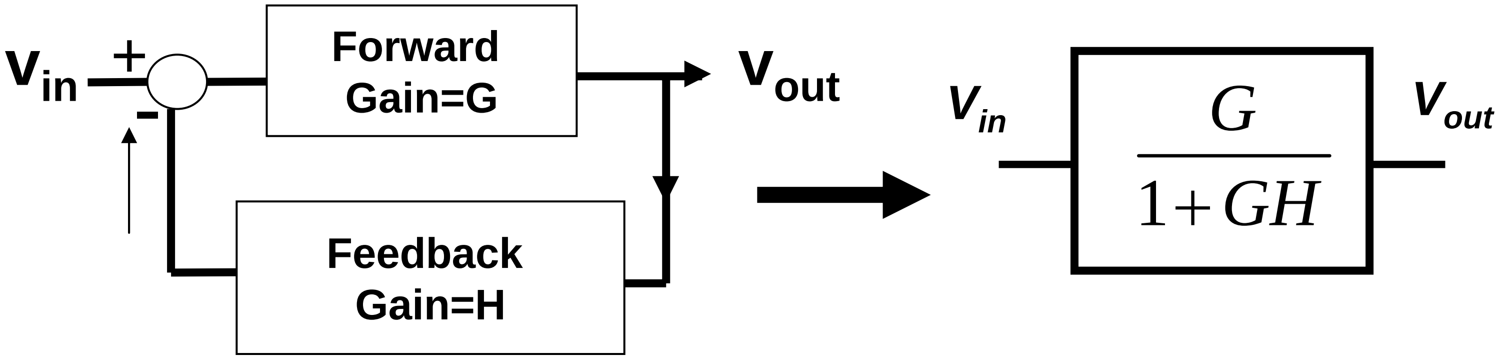
$$v_{out} = G(v_{in} - v_-) \Rightarrow v_{out} = G(v_{in} - Hv_{out}) \Rightarrow v_{out} = \frac{G}{1+GH} v_{in}$$

- *Notice that if G is infinite we recover the familiar result:*

$$\lim_{G\to\infty} v_{out} = \lim_{G\to\infty} \frac{G}{1+GH} v_{in} = \frac{1}{H} v_{in}$$

Modelling the non-inverting amplifier circuit

The non-inverting amp is functionally equivalent to the following block diagram:



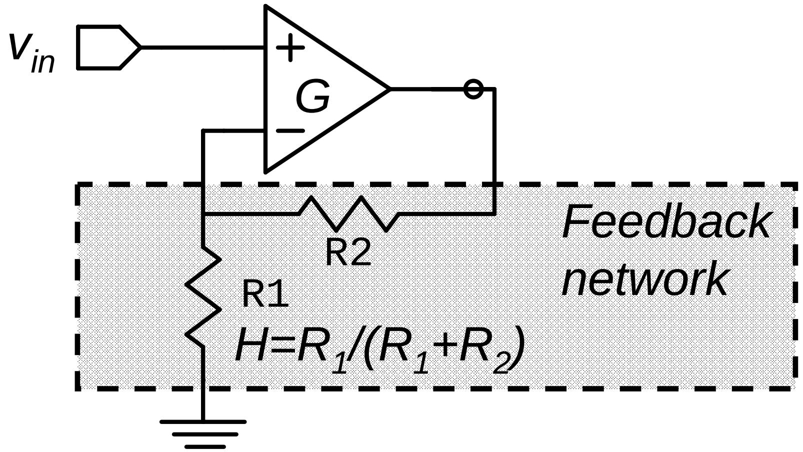
The signals on the network must be self-consistent, so, as before

$$v_{out} = G\left(v_{in} - v_{out}H\right) \Rightarrow v_{out} = \frac{v_{in}G}{1+GH}$$

If GH is large, a Taylor expansion gives: $$\frac{v_{out}}{v_{in}} = \frac{1}{H}\left(1 - \frac{1}{GH} + \frac{1}{(GH)^2} - \cdots\right)$$

- *GH is called the **loop gain** G_L*
- *F=1+GH is sometimes (eg in Franco) called the **feedback factor***
- ***Remember: G and H can be functions of frequency!***

Approximate gain of the non-inverting amplifier



We found that the **closed loop gain** is:

$$v_{out} = G\left(v_{in} - v_{out}H\right) \Rightarrow v_{out} = \frac{v_{in}G}{1+GH} \simeq \frac{v_{in}}{H}\left(1 - \frac{1}{GH} + \cdots\right)$$

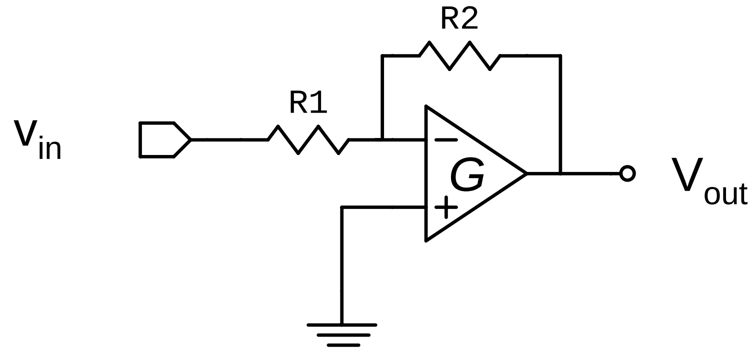
The loop gain GH is usually big when working with monolithic op-amps, so the

gain is approximately the ideal op-amp gain multiplied by: $1 - \frac{1}{GH}$

This Taylor expansion allows us to estimate that we commit a fractional error of (1/GH) if we assume that the op-amp is ideal.

The correction is usually small, especially at lower frequencies, as we will see.

feedback and superposition: the inverting amplifier



By superposition:

$$v_- = v_{in} \underbrace{\frac{R_2}{R_1 + R_2}}_{K} + v_{out} \underbrace{\frac{R_1}{R_1 + R_2}}_{H} = K v_{in} + H v_{out}$$

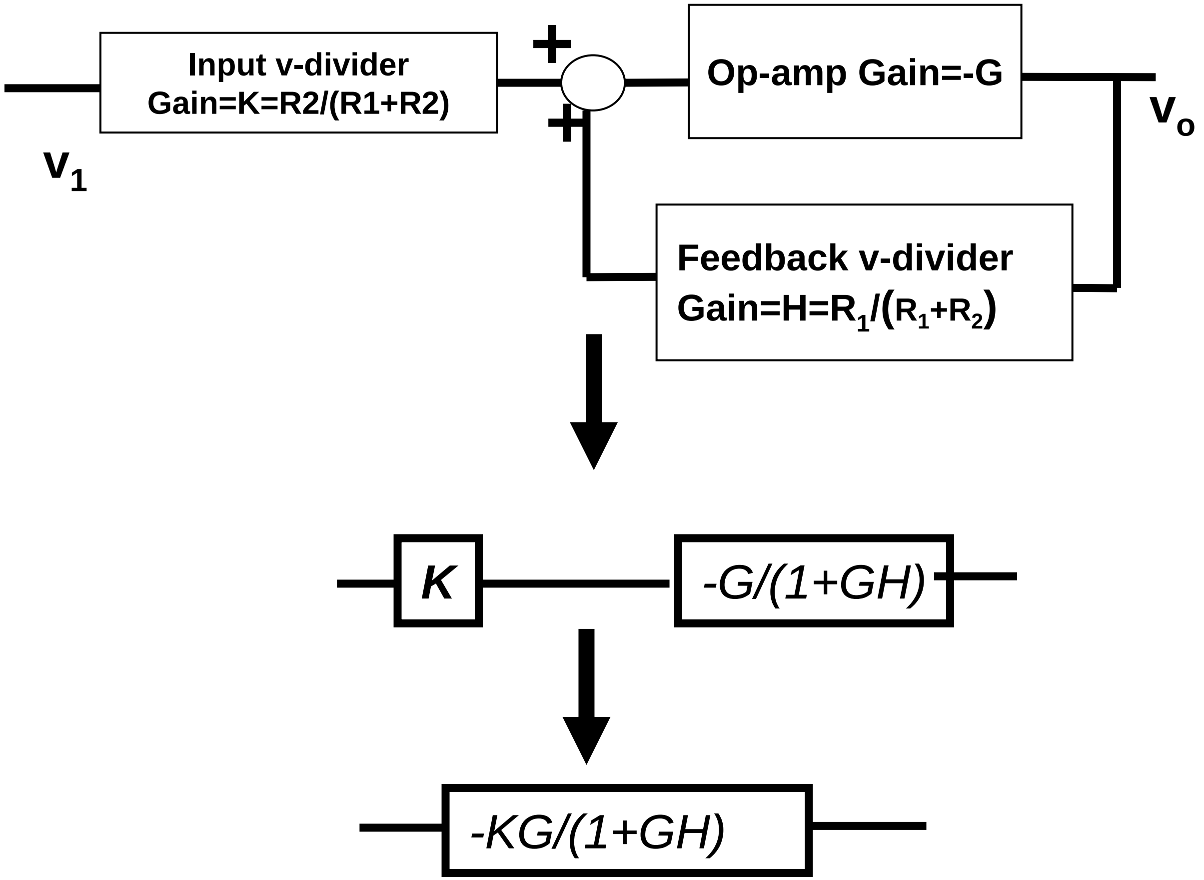
We solve the circuit by requiring it is self-consistent:

$$v_{out} = -G\left(K v_{in} + H v_{out}\right) \Rightarrow v_{out} = \frac{-GK}{1 + GH} v_{in}$$

This is the same result we obtained for the non-inverting amplifier, apart for the extra factor K.
If G is infinite we recover the familiar result:

$$v_{out} = -\lim_{G \to \infty} G\left(K v_{in} + H v_{out}\right) \Rightarrow v_{out} = -\frac{K}{H} v_{in} = \frac{-R_2}{R_1} v_{in}$$

The inverting amplifier:
Calculation using graphs



If ideal op-amp is used:

$$\lim_{G\to\infty} \boxed{-KG/(1+GH)} = \boxed{-K/H}$$

Frequency response of op-amp circuits: The dominant pole approximation

Most monolithic op-amps have the following characteristics:

- **very big,** but not **infinite,** gain at DC
- The **gain is a function of frequency**.
- At lower frequencies op-amps behave roughly like 1st order low pass filters.

Therefore, the gain of an op-amp as a function of frequency is:

$$A(s) = \frac{A_{DC}}{1 + s\tau_0} = \frac{A_{DC}}{1 + j\omega / \omega_0} \quad \omega_0 = 1/\tau_0 = 2\pi f_0$$

- A_{DC} is the **DC gain** of the amplifier, typically $10^4 - 10^6$
- f_0 is the **dominant pole frequency,** typically 10-100 Hz.

The product $A_{DC}f_p$ is called the **gain-bandwidth product (GBW).**

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The gain – bandwidth product

The product $A_{DC}f_p$ is called the **gain-bandwidth product (GBW).**

The GBW is a characteristic constant of the op-amp, typically 10^6-10^8

When we do AC analysis we **must** consider the finite complex gain of the amplifier, especially when we try:

- to get high gain at high frequencies
- to build filters

Any amplifier which can be reasonably accurately described as above is called a “dominant pole amplifier”.

The first order filter description of an amplifier is called the “Dominant Pole approximation”;

it has a remarkable property: The product of gain and bandwidth is constant.

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Invariance of the gain – bandwidth product

Consider a non-inverting amplifier circuit, using a dominant pole amplifier in the forward path. Apply the feedback theory to get the **closed loop gain:**

$$A_V = \frac{G}{1+GH} = \frac{\dfrac{A_{DC}}{1+j\omega/\omega_p}}{1+\dfrac{A_{DC}H}{1+j\omega/\omega_p}} = \frac{A_{DC}}{j\omega/\omega_p+1+A_{DC}H} = \frac{A_{DC}\omega_p}{j\omega+\left(\omega_p+A_{DC}\omega_p H\right)}$$

The DC gain is: $A_{V0} = A_V\left(\omega=0\right) = \dfrac{A_{DC}\omega_p}{\omega_p + A_{DC}\omega_p H}$

The pole (break frequency) of this amplifier is at: $\omega_0 = \omega_p + A_{DC}\omega_p H$

It follows that the product of gain and bandwidth is insensitive to the DC gain:

$$A_{V0}\omega_0 = A_{DC}\omega_p$$

- *This is true for **any** dominant pole amplifier playing the role of the op-amp.*
- *The gain bandwidth product is constant only if the dominant pole approximation adequately describes the amplifier's frequency response.*

•*Super-fast "current feedback op-amps**"** (CFOA) are **not** dominant pole devices. Amplifiers built with CFOA can have a bandwidth which does not depend on gain.*

What is feedback

- We build a circuit to perform an operation f(x) on an input signal. eg amplification, f(x)=Ax
- The circuit behaviour will certainly deviate from what is required.
- What do we do? We build an auxiliary circuit to:
  - *measure the output of the circuit*
  - use the measured quantity to generate an *error signal.*
  - Add the error signal to the input to correct the circuit behaviour
- *How is it done?*
  - Use a voltage or current meter at the output
  - *Apply the* ***inverse*** of the desired function on what we measure
  - Add the measured and processed signal to the input

Feedback in electronics

- There is **both** a voltage and a current at every terminal
- Precise definitions of measurements at the output:
  - **Voltage** is measured **with voltmeters**. Voltmeters are:
    - ***connected in parallel*** *to the circuit*
    - *have infinite input resistance (**Voltage meters draw no current**).*
  - **Current** is measured with **amperometers**. Amperometers are:
    - ***connected in series*** *to the circuit*
    - *have zero input resistance (**current meters develop no voltage**).*
- *Precise definitions of adding the error signal at the input*
  - Voltages are added by connecting voltage sources in series
  - Currents are added by connecting current sources in parallel (“shunt”)

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Feedback connections

- There are 4 ways to implement electronic feedback:
  - **measure** the output:
    - **Voltage**, by connecting the input (port 2!) of the FB network in **shunt** (parallel)
    - **Current**, by connecting the input (port 2!) of the FB network in **series**
  - "**mix**" (feed back) the signal to the input as:
    - **Voltage**, by connecting the output (port 1!) of the FB network in **series**
    - **Current** by connecting the output (port 1!) of the FB network in **shunt** (parallel)

- Feedback topologies are named according to the connections: eg "series-shunt feedback" means series connection at the input and shunt connection at the output
- Exact description of electronic feedback involves 2-port matrix addition; this is very tedious. Most of the time we use approximations like the "Miller Theorem".

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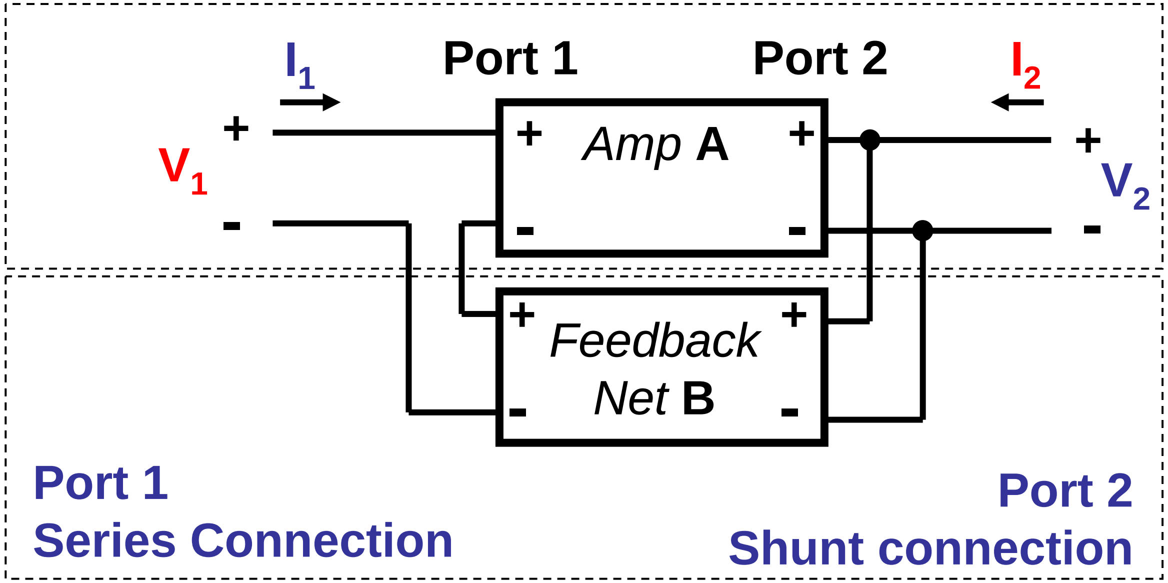
The Series - Shunt connection



Function of the feedback path network:

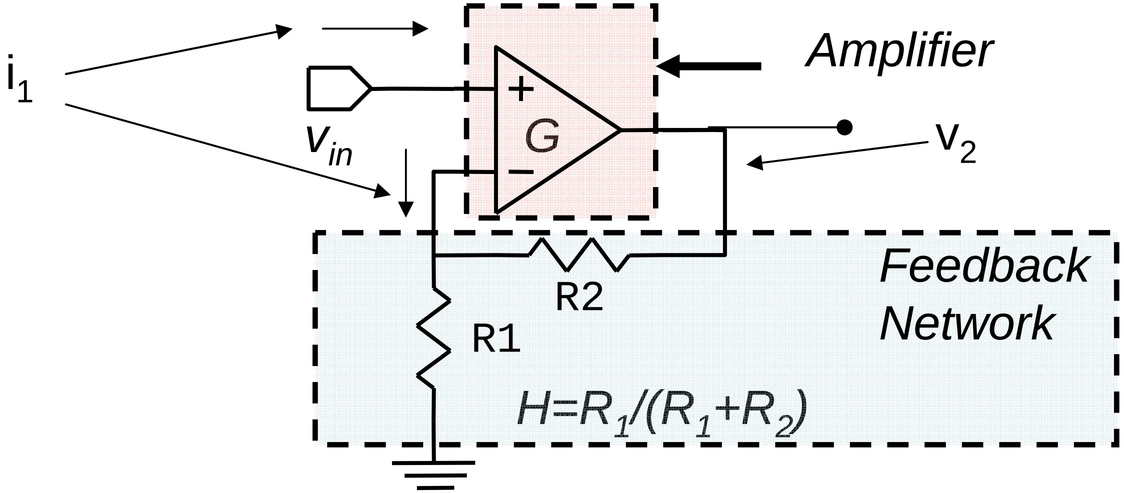
- *Measures the output **Voltage***
- *Feeds back a correction to the input **Voltage** of the forward amplifier*

This connection improves the terminal characteristics of a voltage amplifier (VCVS):

- *Increases the input impedance (bigger voltage for same current)*
- *Decreases the output impedance (bigger current for same voltage)*

The feedback network is functionally a voltage amplifier from Port2 to Port1
Electrically both networks share the electrical variables I1 and V2.

The non-inverting amplifier
an example of series – shunt feedback

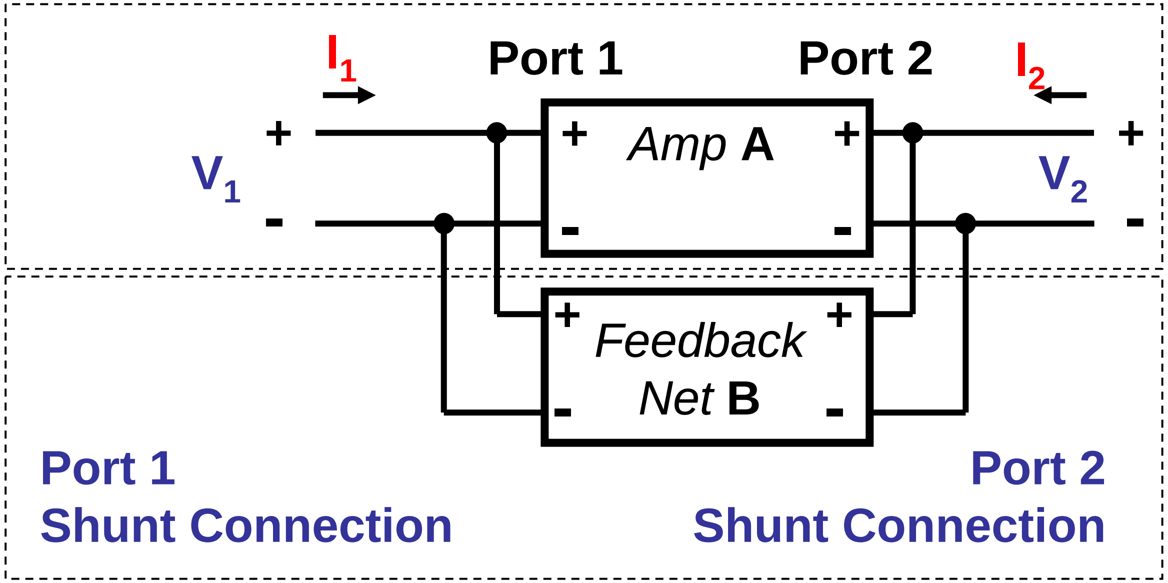


The op-amp acts like a voltage amplifier
The feedback network samples the output voltage, voltage divides it and feeds back a voltage into the input, so that v_{in} is the sum of input and fed-back v.

The feedback network shares with the op-amp (think a finite input impedance!)

- input current I_1 and
- output voltage v_2

The Shunt - Shunt connection



Function of the feedback network:

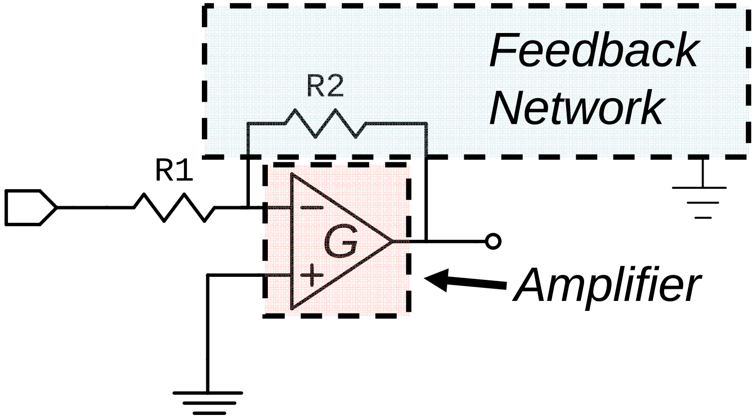
- *Measures the output **Voltage***
- *Feeds back a correction to the input **Current** of the forward amplifier*

This connection improves the terminal characteristics of a transimpedance amplifier (a CCVS):

- *Decreases the input impedance (bigger current for same voltage)*
- *Decreases the output impedance (bigger current for same voltage)*

The feedback network is functionally a transconductance amplifier from Port2 to Port1. Electrically the networks share input and output voltages

The inverting amplifier
An example of shunt – shunt feedback



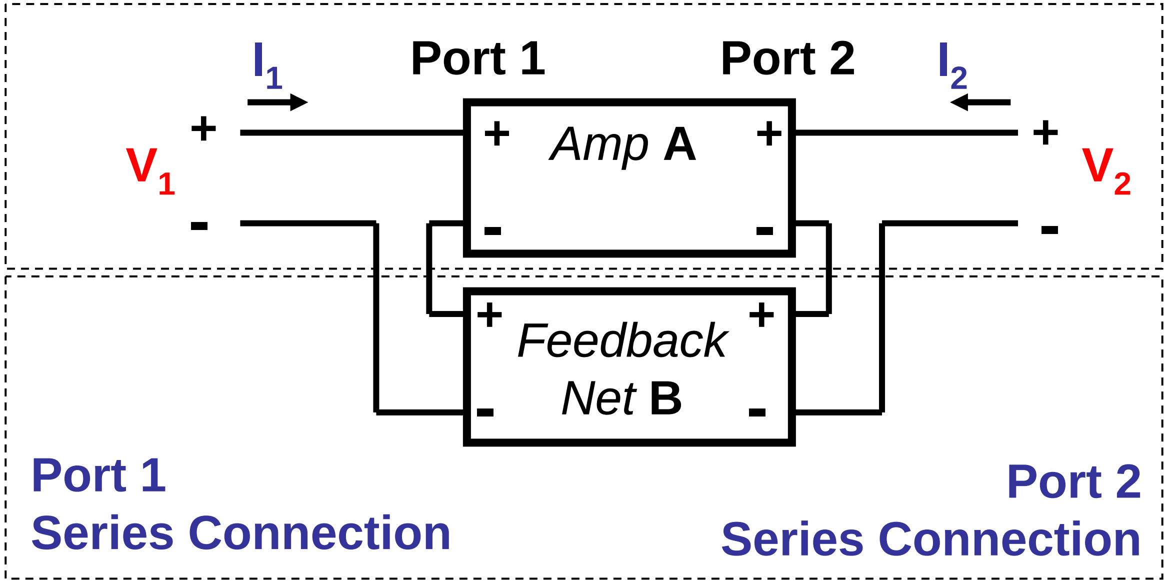
Amplifier and feedback network have same input and output port voltages

Input current to the amplifier is the sum (the “mixture”) of input and feedback path currents. The feedback network samples the output **voltage** and contributes a **current** to correct the input.

The amplifier G functions as a CCVS (but this should not confuse us, we will soon see that the representation is arbitrary!)

This topology is usually solved using Miller theorem!

The Series - Series connection



Function of the feedback path network:

- *Measures the output* ***current***
- *Feeds back a correction to the input* ***Voltage*** *of the forward amplifier*

This connection *improves the terminal characteristics of a transconductance amplifier (VCCS):*

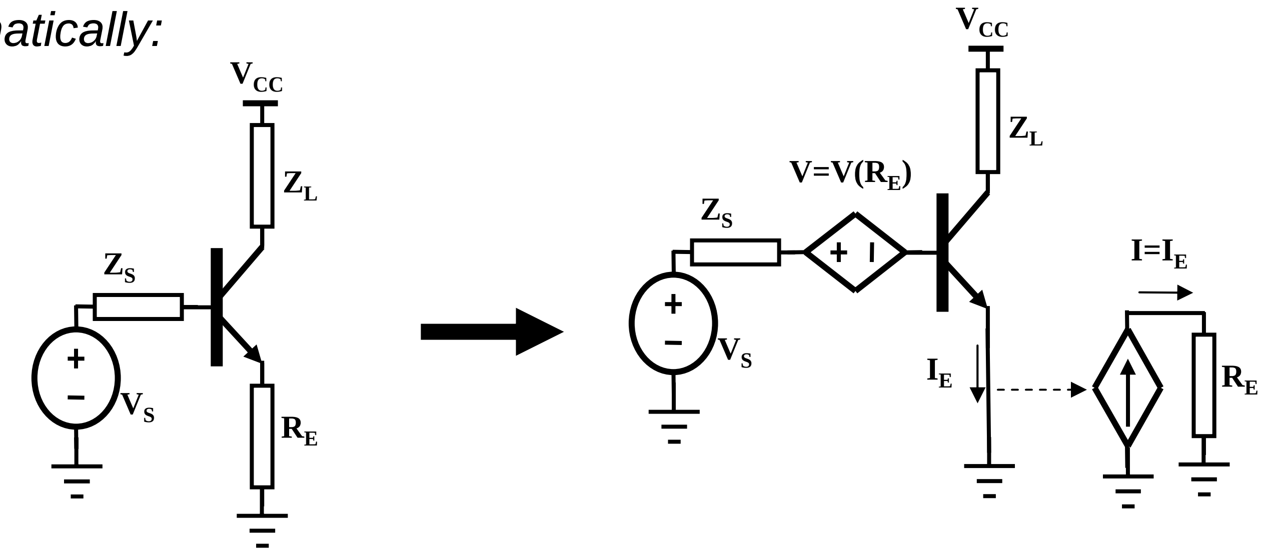
- *Increases the input impedance (bigger voltage for same current)*
- *Increases the output impedance (bigger voltage for same current)*

The feedback network is functionally a transimpedance amplifier from Port2 → Port1
Electrically the two networks share input and output currents.

The emitter-degenerated common emitter amplifier

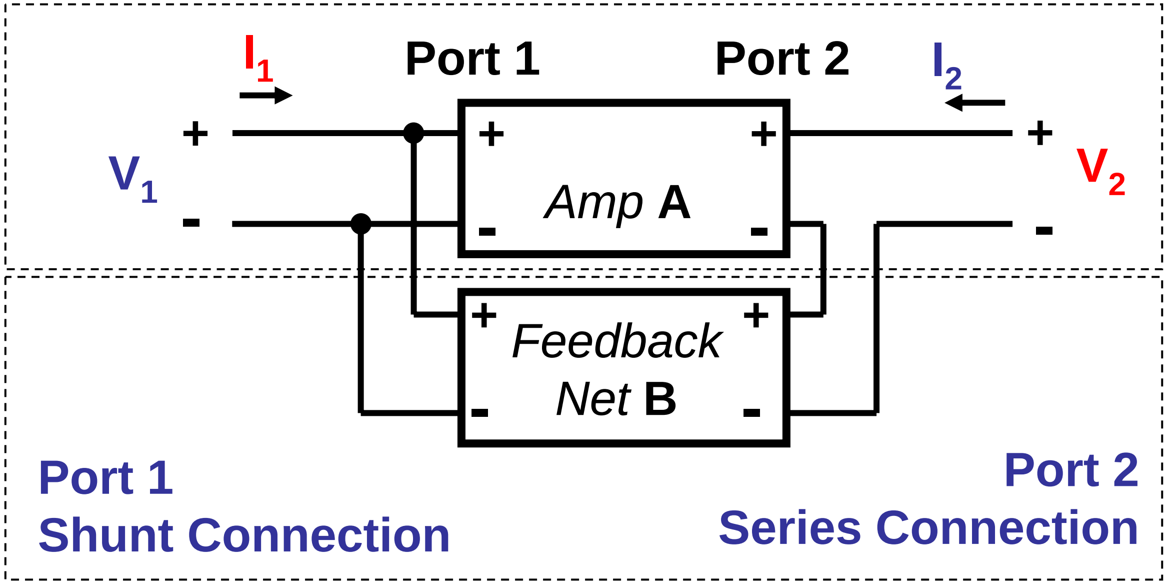
An example of series-series feedback

- *Add a Z_E to the common emitter amplifier*
- *Schematically:*



- *The feedback network samples the emitter current. Since he emitter current is almost equal to the collector current we can say that the feedback network (R_E) samples the output current.*
- *The voltage developed on the feedback resistor is in series with V_{BE} so the feedback voltage is mixed into the input voltage.*
- *This is an example of series-series feedback.*

The Shunt - Series connection



Function of feedback network:

- *Measures the output **Current***
- *Feeds back a correction to the input **Current** of the forward amplifier*

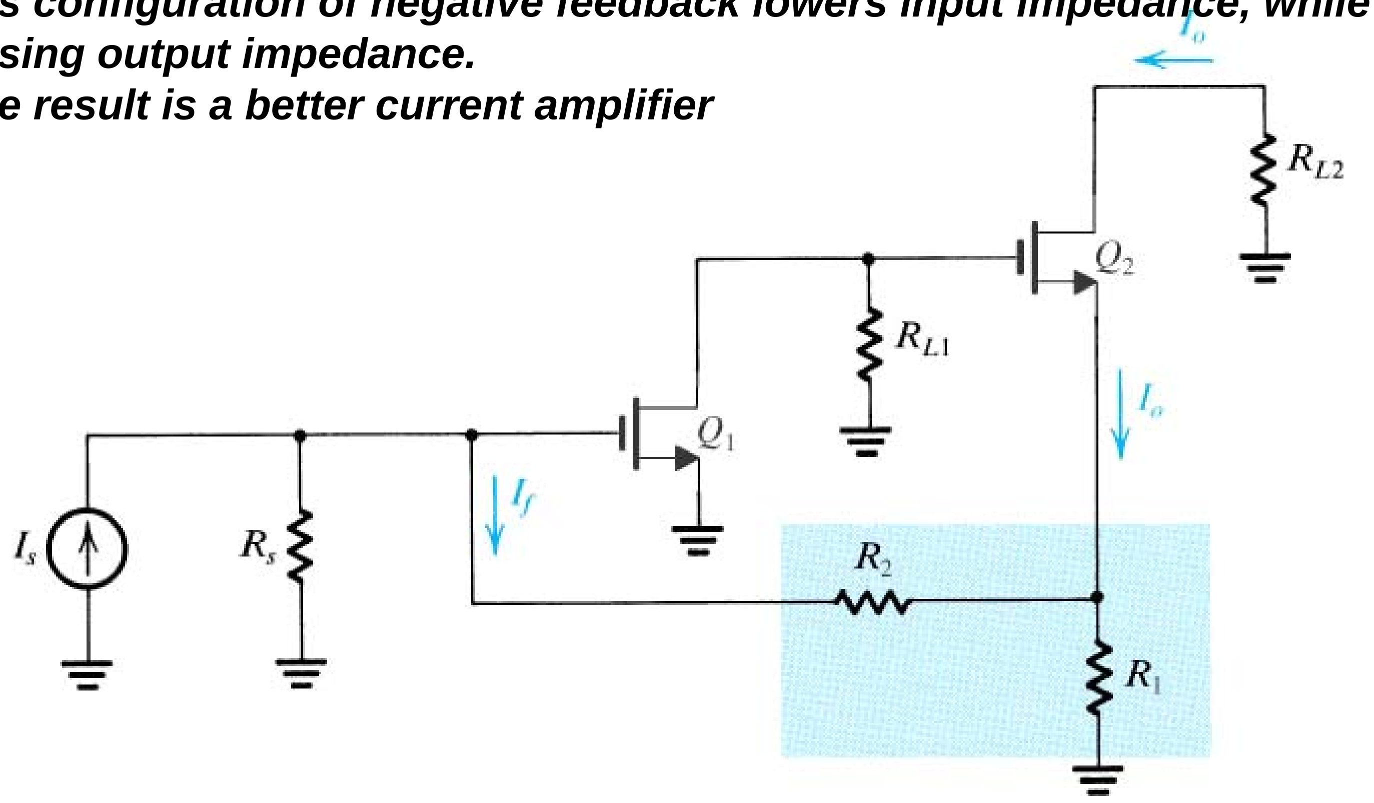
This connection improves the terminal characteristics of a Current Amplifier (CCCS)

- *Decreases the input impedance (bigger current for same voltage)*
- *Increases the output impedance (bigger voltage for same current)*

The feedback network is functionally a current amplifier from Port2 to Port1
Electrically the two networks share V1 and I2.

Shunt – Series feedback

- *Two stage transistor amplifiers allow us to introduce by example the most difficult to understand method of applying negative feedback.*
- *In this example, the output current is sampled and feedback current added to the input current.*
- *this configuration of negative feedback lowers input impedance, while raising output impedance.*
- *The result is a better current amplifier*

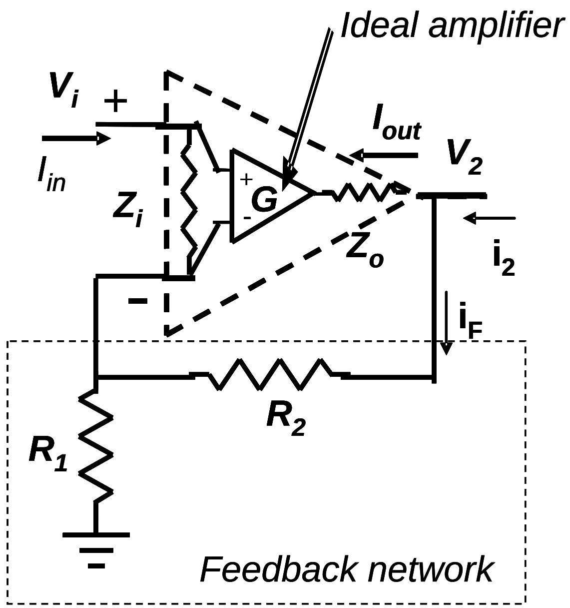


Effect on feedback on input impedance

A realistic op-amp has:

- Finite input impedance
- Finite output impedance
- Finite Gain

Consider a series-shunt connection. The feedback network draws the same input current as the op-amp. This means that:



$$V_i = i_{in} Z_i + V_2 H = i_{in} Z_i + i_{in} Z_i GH = i_{in} \left(1 + GH\right) Z_i \Rightarrow$$

$$Z_{in} = \left(1 + GH\right) Z_i$$

The input impedance appears amplified by a factor (1+GH) (the feedback factor!). provided that the feedback network elements are small compared to Z_i:

$$\left[R_1, R_2\right] \ll Z_i$$

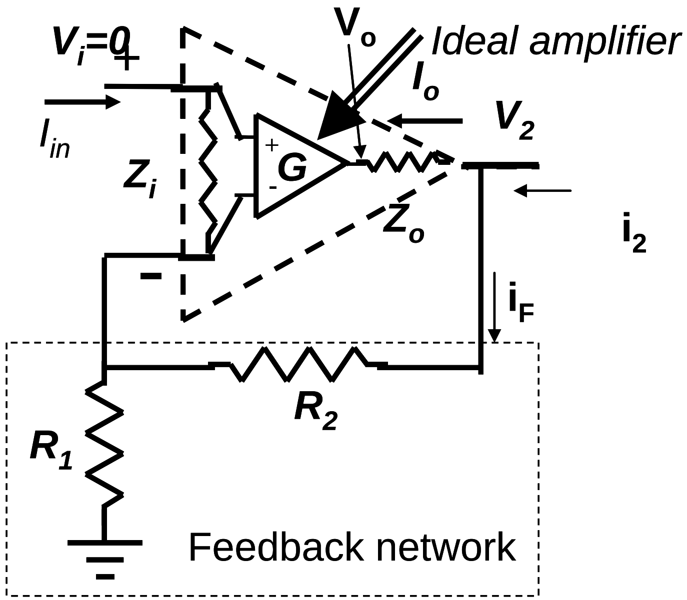
Similarly, a shunt input connection **increases** Y_{in} by a factor 1+GH

Effect on feedback on output impedance

For the output admittance of the closed loop amp:
Consider zero input voltage (i.e. take partial derivative of V_2 with respect to i_2

$$i_{out} = \frac{(V_2 - V_o)}{Z_o} = \frac{(V_2 + Gv_-)}{Z_o} = \frac{V_2(1+GH)}{Z_0} \Rightarrow$$

$$Y_2 = \frac{1+GH}{Z_0}$$

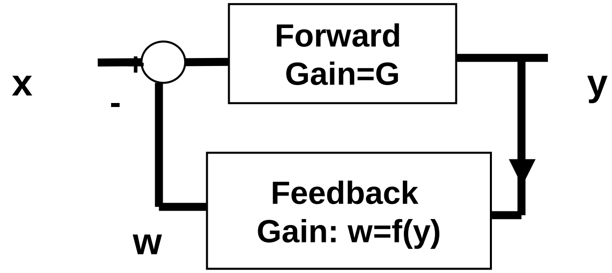


The output impedance in the shunt connection appears reduced by the feedback factor.
This time we have assumed that the feedback network resistors are much larger than the output impedance of the amplifier:

$$Z_o \ll [R_1, R_2]$$

It can be shown that a series output connection **increases** Z_{out} by a factor 1+GH

Nonlinear elements in the feedback path



$$y = G(x - w) = G\left(x - f(y)\right) \Rightarrow x = \frac{y}{G} + f(y) \Rightarrow$$

$$\lim_{G\to\infty} x = f(y) \Rightarrow \lim_{G\to\infty} y = f^{-1}(x)$$

Negative feedback is used to invert functions. Examples:

- Logarithmic amplifiers use BJT or diodes for feedback
- Square root amplifiers use MOSFETs for feedback.

Sensitivity

- Sensitivity is a quantitative answer to questions like:

" What is the % change in gain if the open loop gain of the amp changes by 1%"

By this definition:

$$S_{xy} = \frac{\delta x}{x}\frac{y}{\delta y} = \frac{y}{x}\frac{\partial x}{\partial y} = \frac{\partial \ln x}{\partial \ln y}$$

In other words, the sensitivity is the exponent in a power law dependence.

Example: The sensitivity of a voltage divider ratio on each of the two resistors:

$$G = \frac{R_1}{R_1 + R_2}$$

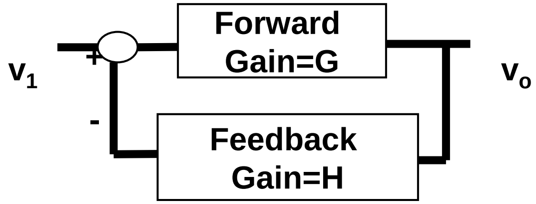
$$S_{G,R_1} = \frac{R_1}{G}\frac{\partial G}{\partial R_1} = \left(R_1 + R_2\right)\frac{R_2}{\left(R_1 + R_2\right)^2} = 1 - G$$

$$S_{G,R_2} = \frac{R_2}{G}\frac{\partial G}{\partial R_2} = \frac{R_2\left(R_1 + R_2\right)}{R_1}\frac{-R_1}{\left(R_1 + R_2\right)^2} = G - 1$$

The result agrees with our intuition that both resistors are equally important: if R_1 increases the ratio increases; If R_2 increases the ratio decreases by the same %. The sensitivity operator is a derivative, and for this reason it obeys a chain rule:

$$S_{y,z} = S_{y,x} S_{x,z}$$

Sensitivity of negative feedback circuits



• *The closed loop gain G_{CL} is the open loop gain G reduced by the "amount of feedback" or "feedback factor" F=1+GH:*

$$G_{CL} = \frac{v_o}{v_i} = \frac{G}{1+GH} = \frac{G}{F}$$

•*The network is less sensitive to variations in G. The* ***% sensitivity*** *of the closed loop gain to % changes in G is again reduced by a factor of F:*

$$\frac{G}{G_{CL}}\frac{\partial}{\partial G}\left(\frac{v_{out}}{v_{in}}\right) = G\frac{1+GH}{G}\frac{\partial}{\partial G}\left(\frac{G}{1+GH}\right) = (1+GH)\frac{1}{(1+GH)^2} = \frac{1}{F}$$

•*The closed loop gain is much more sensitive to variations in H:*

$$\frac{H}{G_{CL}}\frac{\partial}{\partial H}G_{CL} = H\frac{1+GH}{G}\frac{\partial}{\partial H}\left(\frac{G}{1+GH}\right) = \frac{H(1+GH)}{G}\frac{G^2}{(1+GH)^2} = HG_{CL}$$

•*The closed loop gain is more linear. An easy way to see this is to treat non-linearity as a gain variation.*

Summary of the effects of negative feedback

We are given a negative feedback network with forward gain G and feedback path gain H . The system has a loop gain GH and a feedback factor F=1+GH

1.The amplifier gain and its sensitivity to the gain G are both reduced by a factor F

2.For large G the closed loop response approaches the inverse function of H.

3.The closed loop amplifier has input and output impedances related to those of the high gain block G as follows:

- A series connection (S) at a port multiplies the port impedance by a factor F
- A parallel connection (P) at a port multiplies the port admittance by a factor F
- These 2 rules apply both to the input and the output ports.

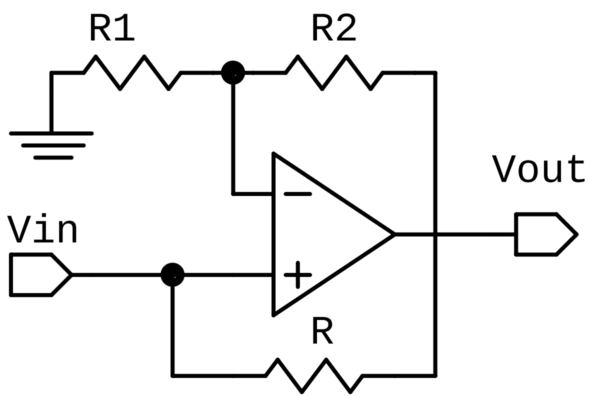
4.The "Miller Theorem" usually simplifies shunt-shunt feedback calculations. We will soon see that there is a second form of the Miller Theorem which simplifies Series-Series connections.

A word of caution: The meaning of "loop gain" depends on the type of connection, but is always dimensionless:

Connection	Forward path gain	Reverse path gain
Series-Shunt	Voltage gain	Voltage gain
Shunt-Series	Current gain	Current gain
Series-series	Transconductance	Transimpedance
Shunt-Shunt	Ttransimpedance	Transconductance

Impedance Arithmetic: Negation and Inversion: The Negative Impedance converter ("NIC")

Note that the op-amp with R_1 and R_2 form an amplifier of gain $G=1+R_2/R_1$
It does not matter that there is **positive feedback** for calculating the port Impedance!



By application of the Miller Theorem:

$$Y_{in} = (1-G)Y_F = \left(1-1-\frac{R_2}{R_1}\right)\frac{1}{R} \Rightarrow Z_{in} = \frac{-RR_1}{R_2}$$

This method is sometimes used to

- *synthesise negative resistances, C's, L's*
- *Invert a given impedance (think of a capacitor in the position of R_1)*
- *Multiply or divide impedance magnitudes (note the ratio R_2/R_1)*

Beware that this circuit may oscillate (as you will learn in Control)

Positive feedback

Same analysis as negative feedback, apart for H → -H

Positive feedback an be used to do things negative feedback cannot do:

- Introduce hysteresis (e.g. Schmitt Trigger)
- Generate negative impedances as we already have seen
- Invert an impedance
- Under positive feedback we can have F=1-GH=0. If F=0 we (in theory) can turn an amplifier into an ideal version by a suitable feedback connection and GH=1.
- However, F=0 at a non-zero frequency is the Barkhausen condition for oscillation.
- Positive feedback is used to make oscillators.

The op-amp is called "**operational**" precisely because it can be used to perform mathematical operations

- *on signals (addition, subtraction, integration, differentiation, multiplication by a scalar,…)*
- *on operators (inversion)*
- *on impedances (negation, inversion, multiplication, division,…)*

Summary

Negative feedback:

- Reduces gain
- Reduces component and environmental sensitivity
- Increases linearity

Positive feedback generally does the opposite of what negative feedback does.

- There are 4 ways to apply electronic feedback
- At the input: ("mix") series or shunt (parallel) connection
- At the output ("sample") series or shunt (parallel) connection
- Feedback can be used to modify input and output impedances:
  – *A series connection multiplies a port impedance by F=1- GH*
  – *A shunt connection multiplies a port admittance by F=1- GH*
- Positive feedback can lead to dynamic instability , i.e. oscillation
- Op-amps are modelled as dominant pole amplifiers:
  – *Have a finite DC gain and a low frequency low pass break frequency ("pole").*
  – *Amplifiers built with op-amps have a constant gain-bandwidth product.*