E2.2 Analogue electronics
Problem sheet 1 - ANSWERS

Q1. Calculate the Thevenin and Norton equivalent circuits of the voltage divider circuit for all the component combinations shown in the table:

\[ Z_T = \frac{1}{Y_N} = \frac{Z_1}{Z_2} + \frac{Z_2}{Z_1} \]

The open circuit voltage is the Thevenin voltage:

\[ V_T = V_0 \frac{Z_2}{Z_1 + Z_2} \]

The Norton current is the short circuit current, i.e.:

\[ I_N = V_T \frac{Z_0}{Z_1} \]

The cases given:

<table>
<thead>
<tr>
<th>V0</th>
<th>Z1</th>
<th>Z2</th>
<th>( V_T )</th>
<th>( Z_T = 1/Y_N )</th>
<th>( I_N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC</td>
<td>Resistor</td>
<td>Resistor</td>
<td>( \frac{V_0 Z_2}{Z_1 + Z_2} )</td>
<td>( \frac{R_1 R_2}{R_1 + R_2} )</td>
<td>( \frac{V_0}{Z_1} )</td>
</tr>
<tr>
<td></td>
<td>Resistor</td>
<td>Capacitor</td>
<td>( \frac{V_0}{1 + j\omega R C} )</td>
<td>( \frac{R_1}{1 + j\omega R C} )</td>
<td>( \frac{V_0}{R_1} )</td>
</tr>
<tr>
<td>AC</td>
<td>Capacitor</td>
<td>Resistor</td>
<td>( j\omega CR V_0 ) ( 1 + j\omega R C )</td>
<td>( \frac{R_1}{1 + j\omega R C} )</td>
<td>( V_0 ) ( j\omega C )</td>
</tr>
<tr>
<td></td>
<td>Resistor</td>
<td>Inductor</td>
<td>( \frac{V_0 i\omega L}{R_1 + j\omega L} )</td>
<td>( \frac{j\omega LR_1}{R_1 + j\omega L} )</td>
<td>( \frac{V_0}{R_1} )</td>
</tr>
<tr>
<td></td>
<td>Inductor</td>
<td>Capacitor</td>
<td>( \frac{V_0}{1 - \omega^2 LC} )</td>
<td>( \frac{j\omega L}{1 - \omega^2 LC} )</td>
<td>( \frac{V_0}{j\omega L} )</td>
</tr>
</tbody>
</table>
Q2. Repeat Q1 but with a current source connected in the place of the voltage source.

**ANSWER:** The impedance $Z_1$ is connected in series to a current source, so it is irrelevant. The Norton equivalent is just the source with $Z_2$.

Q3. Calculate the small signal and the large signal Thevenin and Norton Equivalent circuits of a diode biased with a DC current of 1mA. The saturation current is 1fA. The thermal voltage is 25mV (at 17°C)

**ANSWER:**
Large signal model:
The voltage developed on the diode with 1mA flowing is:

$$V_D = \frac{kT}{q} \ln \left( \frac{I_D}{I_0} \right) = 25mV \cdot \ln \left( 10^{12} \right) = 25mV \cdot 27.6 \approx 0.69V$$

This is also the large signal Thevenin voltage.
The short circuit current is clearly 1mA.
So the large signal equivalent is:
$$V_T = 0.69V, I_N = 1mA, Z_T = 690\Omega$$
The small signal equivalent is again $V_T = V_{OC} = 0.69V$

The small signal Thevenin resistance is quite different than the large signal Thevenin resistance:

$$Y_N = \frac{\partial I_D}{\partial V_D} = \frac{\partial \left( I_0 e^{V_D/kT} \right)}{\partial V_D} = \frac{I_D}{V_T} = \frac{1mA}{25mV} = 40\Omega$$

The Norton current has to be (for consistency!)

$$I_N = \frac{V_T}{Z_T} = \frac{0.69}{40} = 17.25mA$$

This is the case where the small and large signal Thevenin models are quite different!
Q4. A series connection of a diode and a 1kΩ resistor embedded in a big circuit develops 1V DC across it. Calculate the small signal Thevenin resistance of the resistor-diode connection. The diode saturation current is 1fA. The thermal voltage is 25mV (at 17°C) (Do not include the voltage source in the calculation, only the diode and the resistor!)

**ANSWER**

We need to find the voltage and current developed on the diode. So we need to solve:

\[
\frac{I_D R_D}{A} + \frac{kT}{q} \ln \left( \frac{I_D}{I_0} \right) = 1V
\]

This can be done, for example, by iteration:

We assign the voltage B a starting value, eg B=0.5V then we compute A=0.5V and 

\[ I = \frac{V_A}{1k\Omega} \]

We can then compute a new value for the B term:

\[ V_B = \frac{kT}{q} \ln \left( \frac{I}{I_0} \right) \]

and repeat until the two voltages converge

This is done below:

<table>
<thead>
<tr>
<th>Iteration</th>
<th>A</th>
<th>B</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.0005</td>
</tr>
<tr>
<td>2</td>
<td>0.326553</td>
<td>0.673447</td>
<td>0.000327</td>
</tr>
<tr>
<td>3</td>
<td>0.337204</td>
<td>0.662796</td>
<td>0.000337</td>
</tr>
<tr>
<td>4</td>
<td>0.336401</td>
<td>0.663599</td>
<td>0.000336</td>
</tr>
<tr>
<td>5</td>
<td>0.336461</td>
<td>0.663539</td>
<td>0.000336</td>
</tr>
</tbody>
</table>

We can now calculate the small signal Thevenin impedance:

\[
Z_T = 1k\Omega + \frac{25mV}{0.336mA} = 3972\Omega
\]
Q5. In the circuit diagram above, assume both $Z_1$ and $Z_2$ are arbitrary complex impedances and $V_0$ a sinusoidal source of amplitude $V_0$ and at a frequency $\omega$.

1. Derive an expression for the average power $P_{Z2}$ dissipated in $Z_2$.
2. Derive also an expression for the average total power $P_T$ delivered by the source $V_0$ (which is the power dissipated in $Z_1$ and $Z_2$).
3. What is the power delivered to $Z_2$ if $Z_1$ is finite and $Z_2$ is zero?
4. What is the power delivered to $Z_2$ if $Z_1$ is finite and $Z_2$ is infinite?
5. For what value of $Z_2$ is $P_{Z2}$ maximum, if $Z_1$ is given? What is the maximum fraction of $P_T$ that can be delivered to the load?

ANSWER:

a) $\langle P_{Z2} \rangle = \langle \text{Re} IV'_{Z2} \rangle = \langle |I|^2 \rangle \text{Re} Z_2 = \langle |V|^2 \rangle \frac{R_2}{|Z_1 + Z_2|^2} = \langle |V|^2 \rangle \frac{R_2}{(R_1 + R_2)^2 + (X_1 + X_2)^2}$

b) $\langle P_T \rangle = \langle \text{Re} IV' \rangle = \langle |I|^2 \rangle \text{Re}(Z_1 + Z_2) = \langle |V|^2 \rangle \frac{R_1 + R_2}{|Z_1 + Z_2|^2} = \langle |V|^2 \rangle \frac{R_1 + R_2}{(R_1 + R_2)^2 + (X_1 + X_2)^2}$

c) zero ($VZ_2=0$)

d) zero ($IZ_2=0$)

e) maximum of $\langle P_{Z2} \rangle = \langle |V|^2 \rangle \frac{R_2}{(R_1 + R_2)^2 + (X_1 + X_2)^2}$ occurs if $X_2 = -X_1$

Then the maximum of

$\langle P_{Z2} \rangle = \langle |V|^2 \rangle \frac{R_2}{(R_1 + R_2)^2}$ occurs for

$\frac{\partial}{\partial R_2} \langle P_{Z2} \rangle = 0 \Rightarrow \langle |V|^2 \rangle \frac{\partial}{\partial R_2} \frac{R_2}{(R_1 + R_2)^2} = 0 \Rightarrow \langle |V|^2 \rangle \frac{R_1 - R_2}{(R_1 + R_2)^3} = 0 \Rightarrow R_1 = R_2$

This is $\frac{1}{2}$ of $P_T$. 