E2.2 Analogue electronics  
Problem sheet 2 (Week 4)  

Q1: Determine the following small signal model parameters of an NPN bipolar transistor biased connected as a Common Emitter amplifier and biased with a DC collector current of 1mA and 10mA. The transistor DC current gain is $\beta = 200$ and the Early voltage is $V_A = 100V$:

- Transconductance $g_m$
- Hybrid Pi input resistance $R_\pi$
- Collector output resistance $R_{CE}$

**Answer:**

\[
g_m = \frac{I_C}{V_{th}} = \begin{cases} 
1mA, & g_m = \frac{1mA}{20mV} = 40mS \\
10mA, & g_m = \frac{10mA}{20mV} = 400mS 
\end{cases}
\]

\[
R_\pi = \frac{\beta}{g_m} = \begin{cases} 
1mA, & R_\pi = \frac{200}{40mS} = 5k\Omega \\
10mA, & R_\pi = \frac{200}{400mS} = 500\Omega 
\end{cases}
\]

\[
R_{CE} = \frac{V_A}{I_C} = \begin{cases} 
1mA, & R_{CE} = \frac{100V}{1mA} = 100k\Omega \\
10mA, & R_{CE} = \frac{100V}{10mA} = 10k\Omega 
\end{cases}
\]

Q2: Repeat question 1 if the transistor is PNP but all its other specifications are the same with those of the NPN transistor in question 1.

**Answer:** Same answers, nothing depends on whether the transistor is NPN or PNP

Q3: Draw the small signal equivalent circuit for an NPN bipolar transistor, including capacitances. Label all components. Assume the transistor is properly biased, and is connected as a current amplifier: The emitter is grounded, the base is driven by a current source and the collector is connected to the power supply through a current meter.

Calculate the small signal current gain of this amplifier as a function of frequency. Express this gain as a function of the equivalent circuit elements. Calculate the current gain at low and at high frequencies. At what frequency does the current gain start being frequency dependent?
Answer:

\[
\begin{align*}
    i_B &= \frac{V_{BE}}{R_\pi} \frac{g_m V_{BE} (1 + j \omega R_\pi (C_{BE} + C_{CE}))}{1 + j \omega R_\pi (C_{BE} + C_{CE})} \\
    i_C &= g_m V_{BE} \\
    \beta(\omega) &= \frac{i_C}{i_B} g_m V_{BE} (1 + j \omega R_\pi (C_{BE} + C_{CE})) = \frac{\beta_0}{1 + j \omega R_\pi (C_{BE} + C_{CE})}
\end{align*}
\]

At low frequencies the current gain is essentially constant and equal to $\beta_0$. At high frequencies the dependence is:

\[
\beta(\omega) = \frac{\beta_0 g_m V_{BE}}{\omega (C_{BE} + C_{CE})} = \frac{\beta_0}{\omega R_\pi (C_{BE} + C_{CE})} = \frac{-j \omega}{\omega R_\pi (C_{BE} + C_{CE})}
\]

The transition between the two behaviours happens at

\[
\omega R_\pi (C_{BE} + C_{CE}) = 1 \Rightarrow \omega = \frac{1}{R_\pi (C_{BE} + C_{CE})} = \frac{g_m}{\beta_0 (C_{BE} + C_{CE})} = \frac{\omega_T}{\beta_0}
\]

**Q4:** Determine the $C_{BE}$ and $C_{BC}$ of an NPN bipolar transistor which has an $f_T = 1$ GHz at a bias current $I_C = 1 mA$ if $C_{BE} = 9 C_{BC}$.

**Answer:**

\[
\omega_T = \frac{g_m}{C_{BE} + C_{CE}} = \frac{40 mS}{10C_{CE}} = 2 \pi \times 10^9 \Rightarrow C_{CE} = 636 fF \Rightarrow C_{BE} = 5.73 pF
\]

**Q5:** Draw a diagram of an NPN bipolar transistor connected as a common Emitter voltage amplifier. The base is driven by an ideal voltage source which includes a DC bias $V_0$ so that the DC collector current is 1 mA, and a small AC component $v$ so that $V_{BE} = V_0 + v$.

- Choose a load resistance $R_C$ so that the DC gain is 200
- Choose a DC Collector supply so that the collector has a symmetric maximum swing. The Saturation voltage is 0.2V
Answer:

The DC gain is $A_v = -g_m R_C \Rightarrow R_c = 200/40 mS = 5 k\Omega$

The DC drop on $R_C$ is $5V$, so the supply needs to be $V_{CC} = V_{SAT} + 2V_{RC} = 10.2V$

Q6: Use the Miller Theorem to calculate symbolically the input impedance of an inverting amplifier built with an op-amp whose open loop gain is $G=10$.

Answer:

Since the op-amp gain is $G=10$, it follows that the input impedance is $Z_{in} = R_1 + R_2 / (G+1) = R_1 + R_2 / 11$

Q7: Draw a small signal model of the circuit in Question 5. Use the Miller Theorem to simplify the circuit. Assume that for this circuit the results of Question 4 are valid. Calculate the frequency response of the voltage gain of this amplifier in the following 2 cases:

- The signal source has a Thevenin impedance $R_t = 0$
- The signal source has a Thevenin impedance $R_t = 1 k\Omega$
\[ R_{s1} = \frac{R_x}{\gamma R_x} = \frac{Z_s}{1 + Z_s/R_x} \]

\[ C_2 = C_{BE} + \left( -A_v + 1 \right) C_{BC} = 9C_{BC} + 201C_{BC} = 210C_{BC} = 1.2 \text{nF} \]

\[ R_2 = \frac{R_{CE}}{R_L} = \frac{R_L}{1 + R_L/R_{CE}} \]

\[ V_{s1} = \frac{V_s}{1 + Z_s/R_x} \]

If \( R_s = 0 \) then
\[ A_v(\omega) = \frac{v_o}{v_s} = -g_m R_2 \frac{1}{1 + j\omega R_2 C_3} \]

If \( R_s \) is finite, then
\[ A_v(\omega) = \frac{v_o}{v_s} = -g_m R_2 \frac{1}{1 + Z_s/R_x} \frac{1}{1 + R_{S1, j\omega R_2 C_3}} \frac{1}{1 + j\omega R_2 C_3} \]