E2.2 Analogue electronics
Problem sheet 5 (Week 8)

An NPN bipolar transistor is given with $C_{BE} = 5\, pF$, $C_{BC} = 1\, pF$, $C_{CE} = 0.1\, pF$, $V_A = 100\, V$, $\beta = 200$. For the purposes of this problem sheet the transistor is biased at $I_C = 1\, mA$

**QUESTION 1**: Use the second form of the Miller Theorem to derive an expression for the input impedance of a common collector amplifier which drives an inductive load $L_E$. Derive an expression for the frequency at which the input impedance of this amplifier becomes real, and a value for the input impedance at that frequency. Neglect the effect of $C_{BC}$.

The calculation will be greatly simplified if you use the following substitutions:

$$R = R_s, C = C_{BE}, L = (\beta + 1) L_E, \tau = RC, \tau_1 = \frac{L}{R}$$

**QUESTION 2**: Calculate the value of the load resistance $R_L$ which results in the maximum voltage gain of a Common Emitter amplifier employing Miller Cancellation as a function of the frequency at which Miller cancellation is implemented. You may assume that the source impedance satisfies $R_s \ll R_L$. Show that in the limit of very high Miller cancellation frequency the maximum obtainable gain varies as $\omega^{-2}$.

**QUESTION 3**: A negative feedback amplifier is built with a dominant pole forward amplifier and feedback gain constant with frequency. Prove that the closed loop amplifier is also a dominant pole amplifier whose gain-bandwidth product is constant and equal to the gain-bandwidth product of the forward amplifier.
QUESTION 4: (Tutorial Question Week 8-9)

Show that the admittance matrix of the “Pi” network shown on the left below is:

\[
\begin{bmatrix}
Y_1 + Y_C & -Y_C \\
-Y_C & Y_2 + Y_C
\end{bmatrix}
\]

Connect an admittance \(Y_C\) between the input and output of a network with the admittance matrix: \(Y_0 = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}\). Use KCL to show that the resulting circuit (shown in the figure below) has an admittance matrix equal to:

\[
Y' = \begin{bmatrix}
y_{11} + Y_C & y_{12} - Y_C \\
y_{21} - Y_C & y_{22} + Y_C
\end{bmatrix}
\]

The admittance matrix of a 2-port is defined as: \(Y_{ij} = \frac{\partial i_i}{\partial v_j}\bigg|_{\delta v_{ij}=0}\) i.e. the current in port “i” is measured by an ideal ammeter as a voltage differential is applied on port “j”.

![Diagram of Pi network](image-url)