4. Amplifiers

Any signal processing (such as amplification, sampling, filtering) adds noise to a measurement. The highest Signal-to-Noise-Ratio in a system is found, therefore, at the sensor terminals. Yet, it is possible to reduce the apparent noise in a measurement. The most common kind of noise is actually interference, which can be reduced by resorting to common mode rejection techniques, as are outlined below. In other situations the signal bandwidth may be much narrower than the system bandwidth in which case filtering may increase the SNR. An amplifier may have its minimum noise factor at a frequency different than the measurement frequency, so it is preferred to modulate the signal, i.e. frequency translate it to a different frequency where the amplifier is quieter. Finally, since signal and noise are always statistically independent statistical correlation techniques can be used to reduce the noise in the output of an instrument. The trade-off is always instrument complexity for quality of measurement, and the more advanced techniques are reserved for when it is impossible to perform a measurement by simpler means.

4.1.1. Instrumentation Amplifiers

Basic instrumentation amplifier

The term instrumentation amplifier appears with several different meanings in electronics. Its correct meaning is probably a difference amplifier, which ideally has an output proportional to the difference of two signals applied to it. Its purpose in the laboratory is to subtract from a signal a known interference background. Measuring the imbalance signal of a bridge is such an application. A practical circuit which approximates the ideal differencing amplifier is:

\[ V_o = V_{i+} \frac{R_3}{R_3 + R_4} - V_{i-} \frac{R_4}{R_3} \] (1)

so that if \( R_1R_4 = R_2R_3 \) we can write:

\[ V_o = G (V_{i+} - V_{i-}) \text{, } G = \frac{R_2}{R_3} = \frac{R_4}{R_3} \] (2)

G is the differential gain of the amplifier. The elementary difference amplifier circuit suffers from a relatively low input impedance which is dependent on the choice of resistors. The input impedance is in general is unequal between the inverting and non-inverting terminals. The input impedance can be augmented by the addition of unity gain amplifiers at each of the inputs (Figure 2). Both these amplifiers have a pretty poor common mode rejection ratio.
Any pair of signals \( \{x_1, x_2\} \) can be decomposed in terms of the even and add modes:

\[
x_e = \frac{x_1 + x_2}{2}, \quad x_o = \frac{x_1 - x_2}{2}
\] (3)

The mapping is the inverse of itself.

 Ideally we want \( V_{\text{out}} = 0 \) if \( V_{i+} = V_{i-} \). However, in general the output of a difference amplifier is given by:

\[
V_o = G_{\text{cm}} \left( \frac{V_{i+} + V_{i-}}{2} \right) + G_{\text{diff}} \left( V_{i+} + V_{i-} \right).
\]

From (1) we can easily derive \( G_{\text{cm}} \) and \( G_{\text{diff}} \). If \( V_o = AV_{i+} + BV_{i-} \), it follows that:

\[
A = 2G_{\text{cm}} + G_{\text{diff}}, \quad \text{and} \quad B = 2G_{\text{cm}} - G_{\text{diff}} \Rightarrow
\]

\[
G_{\text{cm}} = \frac{A + B}{4}, \quad G_{\text{diff}} = \frac{A - B}{2}.
\]

By direct comparison with (1) we conclude that the common mode gain is:

\[
G_{\text{cm}} = \frac{1}{4} \left( \frac{R_2}{R_3} \frac{R_3 + R_4}{R_2 + R_4} - \frac{R_4}{R_5} \right) = \frac{1}{4} \frac{R_2 R_3 - R_4 R_1}{R_3 (R_1 + R_2)} \quad (4)
\]

and the differential gain is:

\[
G_{\text{diff}} = \frac{1}{2} \left( \frac{R_2}{R_3} R_1 + \frac{R_3}{R_2} R_4 \right) = \frac{1}{2} \frac{R_2 (R_3 + R_4) + R_4 (R_1 + R_2)}{R_3 + R_2} \quad (5)
\]

The ratio of Common mode and differential gains is called the Common Mode Rejection Ratio (CMRR). The CMRR

\[
CMRR = \frac{G_{\text{diff}}}{G_{\text{cm}}} = 2 \frac{R_2 (R_3 + R_4) + R_4 (R_1 + R_2)}{R_2 R_3 - R_4 R_1} \quad (6)
\]

CMRR needs to be large in instrumentation applications where the desired signal may ride on interference signals several orders of magnitude larger.

To get a sense of what (6) means, we allow that nominally \( R_1 = R_2 = R_3 = R_4 \), within a specified tolerance \( \delta \) so that, for example, \( R_1 = R_0 (1 + \delta) \). Then, using standard error propagation rules we get:

Figure 2: A buffered 3 op-amp difference or instrumentation amplifier.
For a 1% component tolerance this evaluates to approximately 49dB, which is very small compared to a typical requirement of CMRR>80dB. Of course, the CMRR can be maximised by trimming $R_2$. (Laser trimming in the case of integrated difference amplifiers).

The CMRR can be further increased by the configuration of Figure 3, the 3-OP-AMP instrumentation amplifier, in which we give the buffers some gain. The common mode gain of the second stage is as before, but the differential arrangement in the buffers, with an implied ground at the symmetry plane, has a unity common mode gain. The CMRR is increased by a factor equal to the buffer gain

$$G_B = 1 + \frac{2R_5}{R_7} = 1 + \frac{2R_6}{R_7}$$

(8)

Note that this configuration allows for a single resistor ($R_7$) adjustment of the differential gain. Indeed, the entire circuit of Figure 3 without $R_7$ is manufactured as an integrated difference amplifier.

**Figure 3:** Standard 3 op-amp instrumentation amplifier with increased CMRR. $R_5 = R_6$ and $R_1R_3 = R_2R_4$.

**Differential Filtering**

The implied ground technique used to build the 3-op-amp instrumentation amplifier is a very powerful one, and is used extensively to perform signal processing in differential signals. For example, to low pass filter a differential signal one would be tempted to use the circuit in Figure 4a. However, component mismatch would lead to slightly different time constants for the two circuits:

$$\tau_+ = R_v C_+ = \tau (1+\epsilon)$$

$$\tau_- = R_- C_- = \tau (1-\epsilon)$$

(9)

so we would have the following common mode and differential transfer functions:

$$V_{o+} - V_{o-} = \frac{V_{i+}}{1+s\tau(1+\epsilon)} - \frac{V_{i-}}{1+s\tau(1-\epsilon)} = \frac{(V_{i+} - V_{i-})(1+s\tau) - s\epsilon\tau(V_{i+} + V_{i-})}{(1+s\tau(1+\epsilon))(1+s\tau(1-\epsilon))}$$

(10)

From (10) the CMRR appears to be
This means that there is a high pass deterioration of the CMRR. On the other hand, the circuit on Figure 4b has the following solution (by superposition)

\[
V_{o+} = V_{i+} \frac{R_+ + 1/sC}{R_+ + R_- + 1/sC} + V_{i-} \frac{R_-}{R_+ + R_- + 1/sC}
\]
\[
V_{o-} = V_{i+} \frac{R_-}{R_+ + R_- + 1/sC} + V_{i-} \frac{R_+ + 1/sC}{R_+ + R_- + 1/sC}
\]
\[
V_{o+} + V_{o-} = V_{i+} \frac{2R_- sC + 1}{1 + (R_+ + R_-) sC} + V_{i-} \frac{2R_+ sC + 1}{1 + (R_+ + R_-) sC}
\]
\[
V_{o+} - V_{o-} = V_{i+} \frac{1}{1 + (R_+ + R_-) sC} - V_{i-} \frac{1}{1 + (R_+ + R_-) sC}
\]

The CMRR of the amplifier in Figure 4b has the following form:

\[
CMRR \propto \frac{1}{2R_+ sC + 1} \approx \frac{1}{2R_- sC + 1}
\]

The CMRR is now a low pass function, like the differential gain. By introducing an implied ground plane we neutralised some of the effects of component mismatch.

**Figure 4:** Input (low pass) filtering. By observing that the ground level is irrelevant for differential signals we arrive at the differential form on the right, saving 1 capacitor in the process.

Component mismatch is still an issue undermining CMRR even when full differential design is employed. If the dynamic range of the amplifier permits it we can use filtering in the feedback path as shown in Figure 5, where a low pass filter in the feedback path is used to implement an overall high pass filter transfer function. Since a single filter is used, component sensitivities can only alter the filter function characteristic, and not the CMRR. This technique is known as ‘**DC restoration**’. The DC signal from the input is essentially stripped by virtue of the integrator, and a new dc level is restored at the output.
4. Amplifiers

4.1.2. Chopper Amplifiers

Amplifier drift, which is, really, flicker noise at extremely low frequencies is caused by small changes in the input dc operating point. This input drift is amplified and passed to the output. If the input signal frequency is fairly high then the simple solution is to use an ac-coupled amplifier (a high pass filter) to reject any dc drift. However if the input signal contains a slowly-varying dc component, ac coupling cannot be used since the input dc level would be lost. An extremely stable (drift-free) amplifier is required. A chopper-stabilised amplifier achieves this function by sampling (chopping) the input signal thus converting it to AC. In more detail the chopper amplifier creates an AM signal using the chopping frequency as a carrier. A high pass or a band pass amplifier can then be used to amplify the AM signal, which is now narrowband around the carrier frequency, and less susceptible low frequency flicker noise. After amplification the signal is demodulated and converted back to DC by using a second chopper (modulator) synchronised to the first one.

**Figure 5:** DC restoration in an amplifier. The filtering is applied in a single path and component match is not an issue.

**Figure 6:** Principle of a chopper amplifier. Instead of amplifying a signal we amplify a modulated carrier and then demodulate.

The bandwidth of the input signal must satisfy the Nyquist criterion, \(2B < f_s\) to avoid aliasing. Anti-aliasing filters are required with chopper amplifiers as shown on Figure 6.

The concept of a chopper amplifier can be generalised. The carrier does not need to be a pure sine, and the modulation does not even need to be AM. Pseudorandom carriers have been recently proposed for chopper stabilised amplifier. Indeed, any modulator-demodulator link should be possible to use, if it offers an advantage in a given application situation. This, however is a subject for signal processing, and we only need to be aware of these possibilities.
The chopper amplifier reduces noise. It averages the input referred flicker noise of the amplifier much in the same way the chopper amplifier examined below.

**Figure 7:** (a) Chopper amplifier noise PSD (b) Input referred noise for non-chopped and chopped amplifier (after Enz and Temes)

### 4.1.3. Goldberg Amplifier

A chopper stabilised amplifier is subject to the sampling theorem spectral restrictions. The carrier (the chopper frequency) needs to exceed the signal bandwidth by at least a factor of 2. This limitation can be overcome by using two separate amplifiers, a chopper amplifier for low frequency and a continuous time amplifier for higher-frequency signals. This is the idea behind the Goldberg amplifier of Figure 8.

![Goldberg amplifier](Image)

**Figure 8:** Goldberg amplifier. The lower frequencies are handled by the chopper amplifier. A high pass filter may be used in the direct signal path.

Assuming the chopper amplifier has a relatively flat frequency response, the low frequency path has a frequency response dominated by the anti-alias filter (LPF). As an example, assume a first order transfer function for the anti alias filter. The gain of the low frequency path is:

\[ H_{LPP} = \frac{-A_1}{1 + s\tau_1} \]

We further assume that the main amplifier is a dominant pole amplifier, so that its gain is:

\[ G_{main} = \frac{A_2}{1 + s\tau_2} \]

It follows that the overall gain is (by superposition):

\[ G = \frac{-A_2}{1 + s\tau_2} \left( 1 + \frac{A_1}{1 + s\tau_1} \right) = -A_2 \left( 1 + \frac{A_1}{1 + s\tau_2} \right) \left( 1 + \frac{A_1}{1 + s\tau_1} \right) = -A_2 \left( 1 + A_1 \right) \frac{1 + s\tau_3}{(1 + s\tau_2)(1 + s\tau_1)} , \quad \tau_3 = \frac{\tau_1}{1 + A_1} \]

if \( \tau_3 = \tau_2 \) then the Goldberg amplifier is a dominant pole amplifier with a gain:

\[ G = \frac{-A_2 \left( 1 + A_1 \right)}{1 + s\tau_1} \]
4.1.4. Autozeroing amplifiers

Many techniques have been invented to minimise drift. They invariably involve some kind of modulation. A popular approach is to commutate a number of amplifiers in and out of the signal path. If two nearly matched amplifiers are available, one can be used to measure its own drift while the other one amplifies the signal, and then, upon exchange of roles the drift measured can be subtracted from the measurement. An old example of such an amplifier is the Harris Commutating Autozeroing Amplifier (CAZ Amp) which achieves drift of less than 0.5µV/year (!) shown in figures 2.21 and 2.22. This IC contains two matched amplifiers which act sequentially to cancel out drift and noise. As one amplifier processes the input signal, the other device stores the null voltage required to cancel out the effects of drift.

Assume that at first Amplifier A1 is active. Capacitor C1 holds the previously stored drift null voltage. Amplifier A2 is connected as a unity gain buffer with input grounded. C2 is thus charged by the offset and noise voltage of amplifier A2. In the next operation phase the internal switches change over, so that A2 becomes active. The capacitor C2 is connected in a direction such as to oppose the offset and noise voltage of amplifier A2. Capacitor C1 is being charged to the drift and noise level of amplifier A1. The amplifiers must switch fast enough so that the drift and noise voltage does not change significantly between the nulling and amplifying phases. The switching frequency must also satisfy the Nyquist criterion. A further design complication arises because the switching frequency must be low enough that the hold capacitors become fully charged during the nulling phase.

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\[ V_{i+} \quad + \quad A_1 \quad + \quad V_o \]
\[ V_{i-} \]

\[ + \quad A_2 \quad - \quad V_{i+} \quad V_{i-} \]

Figure 9: Phase 1 of the autozeroing amplifier operation. Phase (a) A1 amplifies, while A2 samples the offset. Phase (b) the roles are reversed.

Since each amplifier stores and cancels out its own drift voltage, the method does not rely on cancellation between devices. However, the amplifiers should be well-matched in gain since they take turns in amplifying the input signal.

A variation on the CAZ device uses switches at the input to strip off the common-mode signal, leading to CMRR values of better than 100dB.

Note that in principle the autozeroing amplifier uses an analogue memory to store drift and offset voltages. The commutation operations restrict the signal frequencies to several times below the commutation frequency. With typical commutation frequencies in the 100s of Hz, the signal frequencies need to be of the order of 10Hz at most. Autozeroing techniques are generally reserved for measuring slowly varying signals from transducers such as thermocouples, so that the frequency limitation is generally not a problem.

The AZ amplifier has an interesting effect on noise. The subtraction of successive samples is a high pass filtering operation eliminating most of the flicker noise. At the same time, the AZ amplifier is a sampled system so it aliases any signal power at frequencies above the Nyquist frequency, including thermal noise. The two effects are shown in figure 10(a). By optimising the
interplay of different noise contributions one can suppress the 1/f noise contributions to a level comparable to the thermal noise, as shown in figure 10(b)

Figure 10: (a) High pass noise transfer function and noise aliasing gain of the autozeroing amplifier. (b) Output noise PSD of an AZ amplifier (after Enz and Temes)

4.2. Interfacing to High Impedance Sources

Many sensors present extremely high output impedance, complicating the measurement process, and severely limiting the possible bandwidth if the amplifier presents any input capacitance. Such devices include photodiodes, piezoelectric transducers and photomultiplier tubes, and are best modelled as current sources, with significant Norton capacitance. It is precisely the source capacitance which complicates the situation. In principle the current source can be used in conjunction with a large impedance and a voltage amplifier (Figure 11a). This configuration has a very low frequency pole at RC. A preferred approach is to use a transimpedance amplifier (Figure 11b) which generally allows for a greater bandwidth, and possibly lower noise. The transimpedance amplifier has its own shortcomings, though. As is typical, assume that the main amplifier is a dominant pole amplifier with a frequency dependent gain:

$$G = \frac{A_0}{1 + s\tau} \quad (15)$$

The transimpedance (current-to-voltage gain) is given by:

$$H(s) = \frac{V_{out}}{I_s} = -R \frac{A_0}{(A_0 + 1)} \frac{1}{s^2\tau RC / (A_0 + 1) + s(\tau + RC) / (A_0 + 1) + 1} \quad (16)$$

Figure 11: (a) High impedance amplifier. (b) transimpedance amplifier

The transimpedance (current-to-voltage gain) is given by:
The transimpedance amplifier turns out to be a severely underdamped second order low pass filter with a natural frequency (i.e. bandwidth) equal to:

\[ f_o = \frac{1}{2\pi} \sqrt{\frac{A_o + 1}{\tau RC}} = \frac{1}{\sqrt{2\pi}} \sqrt{\frac{GBW}{RC}} \]  

(17)

with GBW the gain-bandwidth product of the core amplifier used to make the transimpedance amplifier. The quality factor of this filter is:

\[ Q = \frac{1}{\sqrt{\frac{\tau}{RC} + \frac{RC}{\tau}}} \]  

(18)

The Q factor can be rather large, especially for high gain and high bandwidth amplifiers. If it is important that the amplifier is critically damped, the amplifier must be defined for Q=0.5. This restricts the gain-bandwidth product of the transimpedance amplifier.

The main contribution to noise in this circuit is from the feedback resistor, which sets the gain of the amplifier.

A configuration which results to a much lower noise, but also of a lower bandwidth, relies on a capacitor to close the feedback path. This is called the “Switched Integrator” and it uses a switched capacitor as the feedback resistor. This circuit is shown in Figure 12. Its gain is set by the feedback capacitor, and is easy to show that if \( \tau \) is the switching period:

\[ \frac{V_{out}}{I_{in}} = \frac{1}{\tau C}. \]

Since this is a switched capacitor circuit the signal bandwidth must be much smaller than the Nyquist frequency, namely \( f_{in} \ll \frac{1}{2\tau} \)

![Figure 12: The ‘Switched integrator’ amplifier](image)

### 4.3. Isolation Amps

Isolation amplifiers are designed to pass analog signals between two points which may differ greatly in their DC potential. The high voltage may be there by design, or because of a fault in the system. Differences of hundreds or even thousands of volts are not uncommon. Isolation amplifiers are required by law in safety critical applications, such as where a fault may cause fire or explosion (chemical reactor sensors, fuel tank sensors, etc) or a fault may cause injury or death, such as biomedical instrumentation, especially for use on humans. The precise regulatory requirements on isolated circuits may be extremely stringent. They typically limit the maximum charge, current, voltage and energy that can be delivered, eg to the patient’s body, under the worst conditions.
conceivable failure, such as circuit components shorting out in the most detrimental way. Isolation amplifiers are typically engineered so that they physically self destruct, i.e. they break apart, in case of certain specified types of failure. In this section we only outline the operating principles of some types of isolation amplifiers. The reliability and regulatory compliance engineering of isolation amplifiers would easily be the subject of an entire lecture course.

4.3.1. Capacitive Coupling

Capacitive coupling is an elementary term describing high pass filtering with a transfer function zero at zero frequency (think that the coupling capacitor forms a high pass filter with the input impedance of the amplifier). A carrier is modulated with the signal, and then ac-coupled across the high voltage barrier. A demodulator across the barrier recovers the signal. The capacitors must be able to withstand high dc voltages over long periods of time without breaking down.

4.3.2. Transformer Coupling

In Figure 13 we see an example of a magnetically isolated amplifier. The principle of operation is similar to that of the chopper stabilized amplifier but with a twist. The pulse generator switches the voltage across capacitor C1 to the transformer winding W1, and imposes an AC flux on all secondary windings. The output of the A1 op-amp drives a modulator which imposes an additional flux proportional to the signal on the transformer through winding W2. The W7 winding supplies signal to the output buffer A2, while the winding W6 provides a feedback path to amplifier A1. The two op-amps are supplied by local linear power supplies built fed by W2 and W4 in the case of amplifier A1 and W3 and W5 in the case of amplifier A2.

The barrier on the T1 dotted line on the diagram is a physical barrier. The input and power windings are physically separated from the output power supply and signal windings. No failure can result in input and output windings fusing together.

Figure 13: Magnetically isolated amplifier
4.3.3. Optical Coupling

Signal transmission across a barrier can be accomplished by optical couplers. The example in Figure 14 uses a matched pair of photodiodes and an LED. One of the photodiodes (D₂) couples signal to the output circuit, while the second (D₁) is used for feedback pre-distortion. It is essential the photodiodes are well matched both electrically and in terms of their optical coupling to the LED. The two amplifiers also need to be well matched, since the feedback is applied to a different amplifier than the output is derived from.

The opto-isolated amplifier will always exhibit a coupling error due to photodiode and amplifier mismatch. Note that all specifications of the amplifiers such as offsets, bias currents, drift, temperature coefficient, etc need to be well matched.

The complexity of the arrangement, and the number of semiconductor devices involved makes the optoisolator amplifier very noisy. Optoisolators are often designed to physically break along the barrier line in case of serious faults.

![Figure 14: Optically isolated amplifier](image-url)