

SYSTEMATIC SYNTHESIS METHOD FOR ANALOGUE CIRCUITS – PART II ACTIVE-RC CIRCUIT SYNTHESIS

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ABSTRACT

This paper and its companion papers (subtitled Parts I and III) together present a systematic synthesis method for analogue circuits. This paper illustrates the systematic synthesis of some basic active-RC circuits by means of examples. We show how different requirements necessitate different synthesis techniques and also how choice of technique affects the synthesised circuit. The synthesis method is shown to be a tool with potential to generate novel circuits.

1 INTRODUCTION

In the companion paper (Part I Notation and synthesis toolbox), the foundations for the systematic synthesis method to be applied in this paper were laid down [1]. We introduced a new admittance matrix notation for the ideal nullor using unbounded elements. A theorem for matrices with unbounded elements allowed us to derive admittance matrix representations for all types of controlled sources and impedance converters from their transmission matrix descriptions. The matrices obtained were transformed into a form suitable for synthesis using a pivotal expansion technique leading to both unbounded and bounded forms. Finally, rows and columns of admittance matrices could be scaled and added to other rows and columns provided the source row and column were not in the core matrix. The definitions of core matrix and matrix equivalence and the nullor and matrix notation in the companion paper [1] will be assumed.

In this paper, we apply the synthesis toolbox to derive some known basic active-RC circuits in order to illustrate how the tools can be effectively deployed. The derivation of complex and novel circuits is left for further work. Many of the circuits synthesised are given in [2,3].

2 TRANSCONDUCTANCES AND IMPEDANCE INVERTERS

We illustrate the use of nullators to move admittance elements between columns of the Y-matrix and the use of norators to allow movement between rows as in [4]; this is justified by the conventional technique for analysing nullor networks [5,6]. Consider the Y-matrix of a positive transconductance, or voltage-controlled current source (VCCS) of gain G_m :

$$\begin{bmatrix} 0 & 0 \\ G_m & 0 \end{bmatrix} \equiv \begin{bmatrix} G_m & 0 \\ 0 & 0 \end{bmatrix} \equiv \begin{bmatrix} \overbrace{0} & 0 \\ 0 & G_m \end{bmatrix} \quad (1)$$

Since G_m is positive, the obvious strategy is to move it onto the main diagonal, allowing it to be realised by a grounded positive conductance. Movement to position 1,1 requires a single norator and, for position 2,2, a single nullator, as shown. However, circuits with an unequal number of nullators and norators are not physical. Another problem is that the nullator and norator in eqn 1 impose constraints which conflict with the specification matrix. We therefore add a fresh node 3 and move the G_m term into the 3,3 position:

$$\begin{bmatrix} 0 & 0 \\ G_m & 0 \end{bmatrix} \equiv \begin{bmatrix} \overbrace{0} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & G_m \end{bmatrix} \quad (2)$$

The partition lines show that the core matrix is 2×2 . The corresponding circuit diagram is shown in Fig 2.1(a).

If G_m in eqn 1 is negative, a different strategy is needed [4]. The $-G_m$ term can be identified with one of the two negative elements describing a non-grounded resistor. Arguments similar to the above show that the non-grounded resistor should be connected between fresh nodes 3 and 4; we move $-G_m$ to the 4,3 position:

$$\begin{bmatrix} 0 & 0 \\ -G_m & 0 \end{bmatrix} \equiv \begin{bmatrix} \overbrace{0} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -G_m & 0 \end{bmatrix} \equiv \begin{bmatrix} \overbrace{0} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & G_m & -G_m \\ 0 & 0 & -G_m & G_m \end{bmatrix} \quad (3)$$

$G_m \quad -G_m$

We show in the centre expression in eqn 3 the three indefinite admittance terms (G_m , G_m and $-G_m$) in row 0 and column 0 which are related to $y_{43} = -G_m$. Adding a nullator between nodes 4 and 0 and a norator between nodes 3 and 0, as shown, will allow us to move the $y_{00} = -G_m$ term to the 3,4 position, as shown. Notice that this nullor also allows the two G_m terms to be moved into their correct places to correspond to a non-grounded

conductance between nodes 3 and 4. The circuit is shown in Fig 2.1(b).

The gyrator or positive impedance inverter, has a Y matrix with $y_{12} = G_{m1}$ and $y_{21} = -G_{m2}$. Its synthesis is a combination of the above techniques:

$$\begin{bmatrix} 0 & G_{m1} \\ -G_{m2} & 0 \end{bmatrix} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & G_{m1} & 0 & 0 \\ 0 & 0 & 0 & G_{m2} & -G_{m2} \\ 0 & 0 & 0 & -G_{m2} & G_{m2} \end{bmatrix} \quad (4)$$

The circuit diagram is shown in Fig 2.1(c). Since the nullors form a continuous loop, nullators and norators may be paired in two different ways.

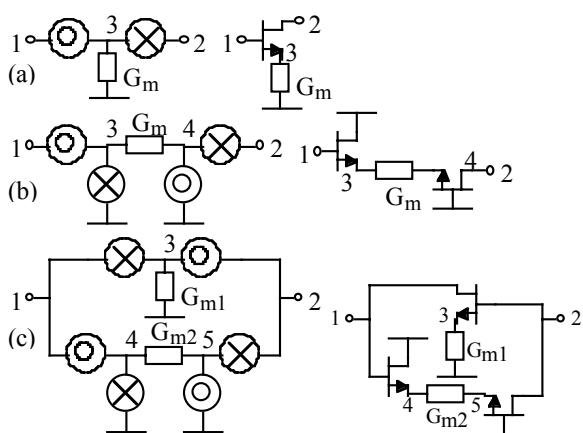


Fig 2.1 Positive and negative transconductors and gyrator

3 GROUNDED AND FLOATING INDUCTANCE

The 1-port grounded simulated inductance must have admittance matrix $[1/(sL)]$, which, for dimensional reasons, we write as $Y = [G_1G_2/(sC)]$. The product and quotient of admittances can be flattened out using a pivotal expansion:

$$\begin{bmatrix} G_1G_2 \\ sC \end{bmatrix} \equiv \begin{bmatrix} - & 0 & G_1- \\ & -G_2 & sC- \end{bmatrix} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & sC & 0 & 0 & 0 \\ \hline 0 & 0 & G_1 & 0 & 0 \\ 0 & 0 & 0 & G_2 & -G_2 \\ 0 & 0 & 0 & -G_2 & G_2 \end{bmatrix} \quad (5)$$

C can be left grounded at node 2; we can move G_1 to the 3,3 position and $-G_2$ to the 5,4 position; finally, we can introduce the remaining G_2 and $-G_2$ terms from row 0 and column 0. Circuit diagrams from this point will not generally be shown as they can be derived easily from the admittance matrix representations.

For the non-grounded simulated inductance, the 2-port admittance matrix together with its pivotally transformed equivalent are as follows:

$$\begin{bmatrix} \frac{1}{sL} & -\frac{1}{sL} \\ -\frac{1}{sL} & \frac{1}{sL} \end{bmatrix} \equiv \begin{bmatrix} G_1G_2 & -G_1G_2 \\ G_1G_2 & -G_1G_2 \end{bmatrix} \equiv \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \equiv \begin{bmatrix} G_1 & \\ & G_1 \\ sC & \\ & sC \end{bmatrix} \quad (6)$$

C can be left grounded at node 3, but we have to move the $\pm G_1$ and $\pm G_2$ terms to suitable positions. The presence of negative terms implies that G_1 and G_2 must both be non-grounded; the final matrix is as follows:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & sC & 0 & 0 & 0 \\ 0 & 0 & 0 & G_2 & -G_2 & 0 \\ 0 & 0 & 0 & -G_2 & G_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_1 \\ 0 & 0 & 0 & 0 & 0 & -G_1 \end{bmatrix} \quad (7)$$

This circuit needs 4 nullors. In order to reduce the number of nullors, we can make use of matrix expansion methods which do not involve introduction of nullors, such as row-column scale-and-add. Subtracting the 3rd column in the pivotally transformed matrix in eqn 6 from the 2nd column, and then subtracting the 3rd row from the 2nd row, we obtain the following:

$$\begin{bmatrix} 0 & -G_1 & G_1 \\ 0 & G_1 & -G_1 \\ -G_2 & G_2 - sC & sC \end{bmatrix} \equiv \begin{bmatrix} 0 & & \\ & G_1 & \\ & G_2 & G_1 + G_2 + sC \\ & G_2 & G_2 + sC \end{bmatrix} \equiv \begin{bmatrix} G_1 & & \\ & G_1 + sC & \\ & G_1 + sC & sC \end{bmatrix} \quad (8)$$

The $\pm sC$ terms and the G_1 term in y_{22} can be left in place. The fact that the other terms tend to share rows and columns to some extent means that only 3 nullors are needed to move them into their required positions; the indefinite admittance terms do not need to be used in this case:

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & G_1 + sC & -sC & -G_1 & 0 \\ \hline 0 & -sC & sC & 0 & 0 \\ 0 & -G_1 & 0 & G_1 & 0 \\ 0 & 0 & 0 & 0 & G_2 \\ 0 & 0 & 0 & 0 & -G_2 & G_2 \end{bmatrix} \quad (9)$$

The circuit needs only 3 nullors.

4 NEGATIVE RESISTANCE

The negative resistance can be described by $Y = [-G]$ and its expansion using 2 nullors is straightforward:

$$[-G] \equiv \begin{bmatrix} 0 & 0 \\ 0 & G & -G \\ 0 & -G & G \end{bmatrix} \quad (10)$$

It is not possible to use one nullor and one conductance, so we let \mathbf{Y} have the form $[-G_1G_3/G_2]$, use a pivotal expansion utilising the 2nd row and 3rd column and subtract column 3 from column 1:

$$\left[\begin{array}{c|c|c} -G_1G_2 & & \\ \hline G_3 & 0 & -G_2 \\ \hline 0 & 0 & 0 \end{array} \right] \equiv \left[\begin{array}{c|c|c} 0 & 0 & -G_1 \\ \hline G_3 & 0 & -G_2 \\ \hline 0 & 0 & 0 \end{array} \right] \equiv \left[\begin{array}{c|c|c} G_1 & 0 & -G_1 \\ \hline G_2+G_3 & 0 & -G_2 \\ \hline 0 & 0 & 0 \end{array} \right] \quad (11)$$

We introduce a nullator to move $G_2 + G_3$ to the 2,2 position and a norator to move the G_1 and G_2 terms from row 0 to row 3:

$$\left[\begin{array}{c|c|c} G_1 & 0 & -G_1 \\ \hline 0 & G_2+G_3 & -G_2 \\ \hline 0 & 0 & 0 \end{array} \right] \equiv \left[\begin{array}{c|c|c} G_1 & 0 & -G_1 \\ \hline 0 & G_2+G_3 & -G_2 \\ \hline -G_1 & -G_2 & G_1+G_2 \end{array} \right] \quad (12)$$

The norator is grounded; the circuit consists of an op-amp and 3 resistors.

5 CURRENT-CONTROLLED VOLTAGE SOURCES

The admittance matrix representation for the CCVS derived in [1] is as follows:

$$\left[\begin{array}{c|c} 0 & G_T \\ \hline \infty_1 & 0 \end{array} \right] \quad (13)$$

where $G_T = R_T^{-1}$. This representation already implies the existence of a nullor. For the positive CCVS, $G_T > 0$; the G_T term can be moved into the 3,3 position by a nullor:

$$\left[\begin{array}{c|c|c} 0 & 0 & 0 \\ \hline \infty_1 & 0 & 0 \\ \hline 0 & 0 & G_T \end{array} \right] \quad (14)$$

For the negative CCVS, $G_T < 0$. The intrinsic nullor allows us to apply the theorem for matrices with unbounded elements, which allows arbitrary finite admittance terms to be added to elements in column 1 and row 2; we let these conductance terms be G_T , $-G_T$ and G_T :

$$\left[\begin{array}{c|c} 0 & -G_T \\ \hline \infty_1 & 0 \end{array} \right] \equiv \left[\begin{array}{c|c} G_T & -G_T \\ \hline -G_T & G_T \end{array} \right] \quad (15)$$

Hence the intrinsic nullor (op-amp) and one resistor are sufficient to realise the negative CCVS.

6 VOLTAGE AMPLIFIERS

In [1], four pivotal expansions A – D were derived for the VCVS of gain $V_2/V_1 = A_v = N/D$. Types C and D include an arbitrary conductance G , which can be exploited in the synthesis. We limit ourselves to a few illustrative examples. Consider an amplifier with positive gain $A_v = G_1/G_2$. Let $N = G_1$ and $D = G_2$ and use expansion C to obtain the following:

$$\left[\begin{array}{c|c|c} 0 & 0 & 0 \\ \hline 0 & 0 & \infty_1 \\ \hline G_1 & -G_2 & G \end{array} \right] \equiv \left[\begin{array}{c|c|c} 0 & 0 & 0 \\ \hline 0 & G_2 & \infty_1 - G_2 \\ \hline 0 & -G_2 & G_2 \end{array} \right] \quad (16)$$

We have let $G = G_2$ and used the theorem for matrices with unbounded elements to add G_2 and $-G_2$ terms in y_{22} and y_{23} ; the last step is to move G_1 to the 4,4 position. The same circuit can be derived by using the unbounded expansion D. Use of expansion B leads to a slightly different circuit with G_2 grounded and G_1 non-grounded.

Consider the VCVS with positive gain $A_v = (G_1 + G_2)/G_2$. Let $N = G_1 + G_2$ and $D = G_2$. Since in this case N includes D , it seems sensible to use an expansion in which N lies on the main diagonal, ie expansion A:

$$\left[\begin{array}{c|c|c} 0 & 0 & 0 \\ \hline \infty_1 & 0 & \infty_1 \\ \hline 0 & \infty_2 & G_1+G_2 \end{array} \right] \equiv \left[\begin{array}{c|c|c} 0 & 0 & 0 \\ \hline \infty_1 & G_2 & \infty_1 - G_2 \\ \hline 0 & \infty_2 & G_1+G_2 \end{array} \right] \quad (17)$$

We have used the theorem to add terms G_2 and $-G_2$ in y_{22} and y_{23} . The circuit is the familiar non-inverting amplifier circuit with an op-amp and 2 resistors. The same circuit may be derived from the unbounded expansion D.

Finally, we consider the VCVS with negative gain of the form $A_v = -G_1/G_2$. Let $N = -G_1$ and $D = G_2$. We use expansion C, let $G = G_1 + G_2$ and use the theorem to add G_2 and $-G_2$ terms in row 2:

$$\left[\begin{array}{c|c|c} 0 & 0 & 0 \\ \hline 0 & 0 & \infty_1 \\ \hline -G_1 & -G_2 & G_1+G_2 \end{array} \right] \equiv \left[\begin{array}{c|c|c} 0 & 0 & 0 \\ \hline 0 & G_2 & \infty_1 - G_2 \\ \hline 0 & -G_2 & G_1+G_2 \end{array} \right] \quad (18)$$

Then we use a nullator to move $-G_1$ at position 3,1 to 3,4 and a norator to bring the $-G_1$ and G_1 terms from row 0 to row 4. The circuit consists of the familiar inverting amplifier circuit with an op-amp and 2 resistors preceded by a voltage follower op-amp.

If the inverting amplifier is allowed to have a non-zero input conductance of G_1 , then the specification matrix becomes:

$$\left[\begin{array}{c|c|c} G_1 & 0 & 0 \\ \hline 0 & 0 & \infty_1 \\ \hline -G_1 & -G_2 & G_1+G_2 \end{array} \right] \equiv \left[\begin{array}{c|c|c} G_1 & 0 & -G_1 \\ \hline 0 & G_2 & \infty_1 - G_2 \\ \hline -G_1 & -G_2 & G_1+G_2 \end{array} \right] \quad (19)$$

We use the theorem to introduce the $-G_1$ and $-G_2$ terms in column 3 and the G_2 term in row 2. The circuit is the familiar single op-amp inverting amplifier circuit.

7 CURRENT AMPLIFIERS

In [1], four pivotal expansions were derived for the CCCS of gain $I_2/I_1 = A_1 = N/D$: First we consider a non-inverting amplifier with positive gain $A_1 = G_2/G_1$, ie $N = G_2$ and $D = G_1$ and we use expansion C to give (eqn 20):

$$\begin{aligned} \left[\begin{array}{cc|c} 0 & 0 & -G_1 \\ 0 & 0 & -G_2 \\ \hline \infty_1 & 0 & G \end{array} \right] &= \left[\begin{array}{cc|cc} G_1 & 0 & -G_1 & 0 \\ 0 & 0 & 0 & 0 \\ \hline \infty_1 & \infty G_1 & 0 & G_1 + G_2 \\ 0 & 0 & \infty G_2 & G_2 \end{array} \right] \\ \equiv G_1 \quad 0 \quad \equiv G_1 & \\ \equiv 0 \quad 0 \quad \equiv G_2 & \\ \equiv \equiv_1 = G_1 \quad 0 \quad G_1 + G_2 & \end{aligned} \quad (20)$$

We have changed the signs of G_1 and G_2 because they are both off-diagonal, we have let $G = G_1 + G_2$ and used the theorem to add G_1 and $-G_1$ terms in y_{11} and y_{31} . Next, we used a norator to move $-G_2$ from the 2,3 to the 4,3 position and finally used a nullator to introduce the G_2 and $-G_2$ terms from column 0. Use of expansion B to realise this function leads to a different circuit with two grounded resistors.

Consider the inverting CCCS with gain of the form $A_1 = -G_2/G_1$. We use expansion C and let $N = G_2$, $D = -G_1$ and $G = G_1$ (eqn 21):

$$\left[\begin{array}{cc|c} 0 & 0 & -G_1 \\ 0 & 0 & G_2 \\ \hline \infty_1 & 0 & G_1 \end{array} \right] \equiv G_1 \quad 0 \quad \equiv G_1 \\ \equiv 0 \quad 0 \quad \equiv G_2 \\ \equiv \equiv_1 = G_1 \quad 0 \quad G_1 \quad \equiv \left[\begin{array}{cc|cc} G_1 & 0 & -G_1 & 0 \\ 0 & 0 & 0 & 0 \\ \hline \infty_1 & \infty G_1 & 0 & G_1 \\ 0 & 0 & 0 & G_2 \end{array} \right]$$

We use the theorem to add G_1 and $-G_1$ in column 1; an additional nullor moves the G_2 term to the 4,4 position.

Finally consider the inverting CCCS with gain $A_1 = -(G_1 + G_2)/G_1$. Use expansion A with $N = G_1 + G_2$, $D = -G_1$:

$$\left[\begin{array}{cc|c} 0 & 0 & -G_1 \\ \infty_1 & 0 & 0 \\ \hline \infty_1 & 0 & G_1 + G_2 \end{array} \right] \equiv G_1 \quad 0 \quad \equiv G_1 \\ \equiv \equiv_1 = 0 \quad 0 \quad \equiv \\ \equiv \equiv_1 = G_1 \quad 0 \quad G_1 + G_2 \quad \equiv \quad (22)$$

We have used the theorem to add terms G_1 and $-G_1$ in column 1; the circuit has one nullor.

8 IMPEDANCE CONVERTERS

Expansions for the impedance converter with voltage gain $A_V = N_V/D_V$ and current gain $A_I = -N_I/D_I$ were given in [1]. Let N_V , D_V , N_I and D_I be defined by conductances $G_1 - G_4$:

$$\left[\begin{array}{cc|cc} 0 & 0 & D_I & 0 \\ 0 & 0 & -N_I & 0 \\ \hline 0 & 0 & 0 & 0 \\ -N_V & D_V & 0 & 0 \end{array} \right] \equiv 0 \quad 0 \quad G_4 \quad 0 \\ \equiv 0 \quad 0 \quad \equiv G_3 \quad 0 \\ \equiv 0 \quad 0 \quad 0 \quad 0 \\ \equiv G_1 \quad G_2 \quad 0 \quad 0 \quad (23)$$

Eqn 23 may be synthesised in the usual way; however, inverting the voltage and current gains leads to a simpler circuit with only 3 nullors:

$$\left[\begin{array}{cc|cc} 0 & 0 & G_4 & 0 \\ 0 & 0 & G_3 & 0 \\ \hline 0 & 0 & 0 & 0 \\ G_1 & G_2 & 0 & 0 \end{array} \right] \equiv \left[\begin{array}{cc|cccc} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & G_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & G_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_2 \end{array} \right] \quad (24)$$

For the NIC, $V_1 = V_2$ and $I_2 = (G_2/G_1)I_1$. We use expansion C in [1], choose D and N to be negative as they are both off-diagonal and choose $G = G_1 + G_2$:

$$\left[\begin{array}{cc|c} 0 & 0 & -G_1 \\ 0 & 0 & -G_2 \\ \hline \infty_1 & \infty_1 & G_1 + G_2 \end{array} \right] \equiv 0 \quad G_1 \quad \equiv G_1 \\ \equiv 0 \quad G_2 \quad \equiv G_2 \\ \equiv \equiv_1 = G_1 \quad \equiv_1 = G_2 \quad G_1 + G_2 \quad \equiv \quad (25)$$

$$\equiv G_1 \quad 0 \quad \equiv G_1 \\ \equiv 0 \quad G_2 \quad \equiv G_2 \\ \equiv \equiv_1 = G_1 \quad \equiv_1 = G_2 \quad G_1 + G_2 \quad \equiv$$

We use row-column scale-and-add to subtract column 3 from column 2. The intrinsic nullator between nodes 1 and 2 allows the G_1 and $-G_1$ terms to be moved to column 1. The circuit is the familiar one with one op-amp and 2 resistors. A similar technique may be used to synthesise the GIC [2]. The negative resistance circuit derived in eqn 12 is the same as that of the NIC derived in eqn 25, but with a grounded resistor G_3 at port 2. However, each circuit was derived from its own specification without any a priori assumption.

9 CONCLUSIONS

We have illustrated the use of a systematic synthesis method for analogue circuits by the synthesis of some known simple active-RC circuits. We observed that different techniques are appropriate for different requirements and that different techniques applied to the same requirement can produce different circuits. Nullors in the synthesised circuits may be replaced by op-amps or transistors. In the third of three papers on systematic analogue circuit synthesis [7] we consider all transistor circuits.

10 REFERENCES

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