Multiple stage amplifiers

Aims:

- *Examine a few common 2-transistor amplifiers:*
  - *-- Differential amplifiers*
  - *-- Cascode amplifiers*
  - *-- Darlington pairs*
  - *-- current mirrors*
- *Introduce formal methods for exactly analysing multiple stage amplifiers*

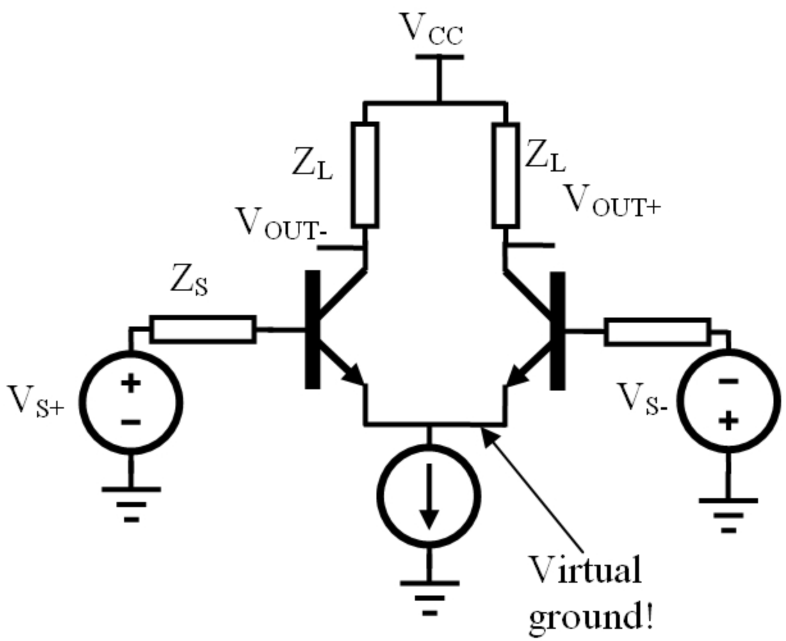
Two stage BJT amplifiers

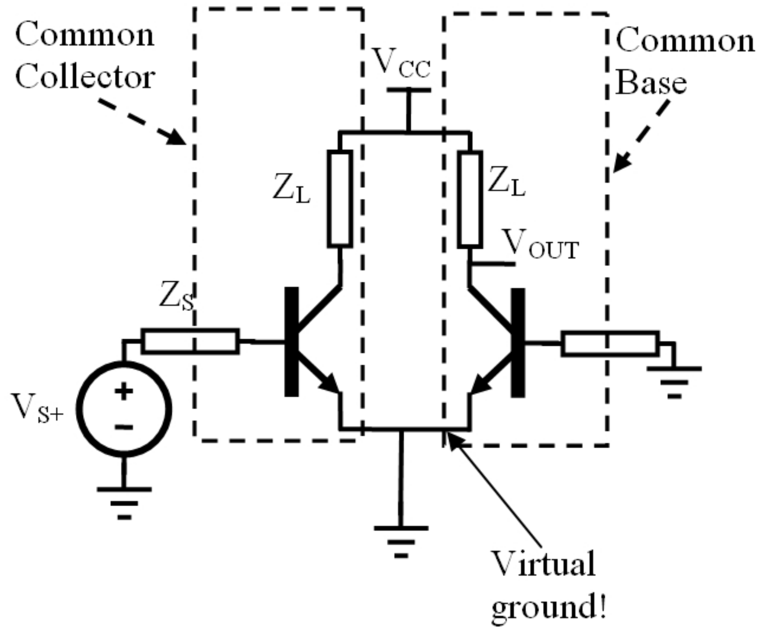
We study them separately because they very often appear as building blocks.
There are 9 possible cascades of 2 single stage transistor amplifiers.
We will study the shaded ones.

	BJT		Comments
Name	1st Stg	2nd Stg	
(voltage amp)	CE	CE	High Voltage gain
cascode	CE	CB	High bandwidth
(op-amp)	CE	CC	High Z_{in} low Z_{out}
(current buffer)	CB	CE	Higher Z_{out} than CB/CG
(current buffer)	CB	CB	Second stage to improve on CB/CG
(Not common)	CB	CC	Not common
(Not common)	CC	CE	Instead of CE, offers higher Z_{in}
differential amp	CC	CB	High voltage gain and bandwidth
darlington	CC	CC	High current gain

Differential amplifier

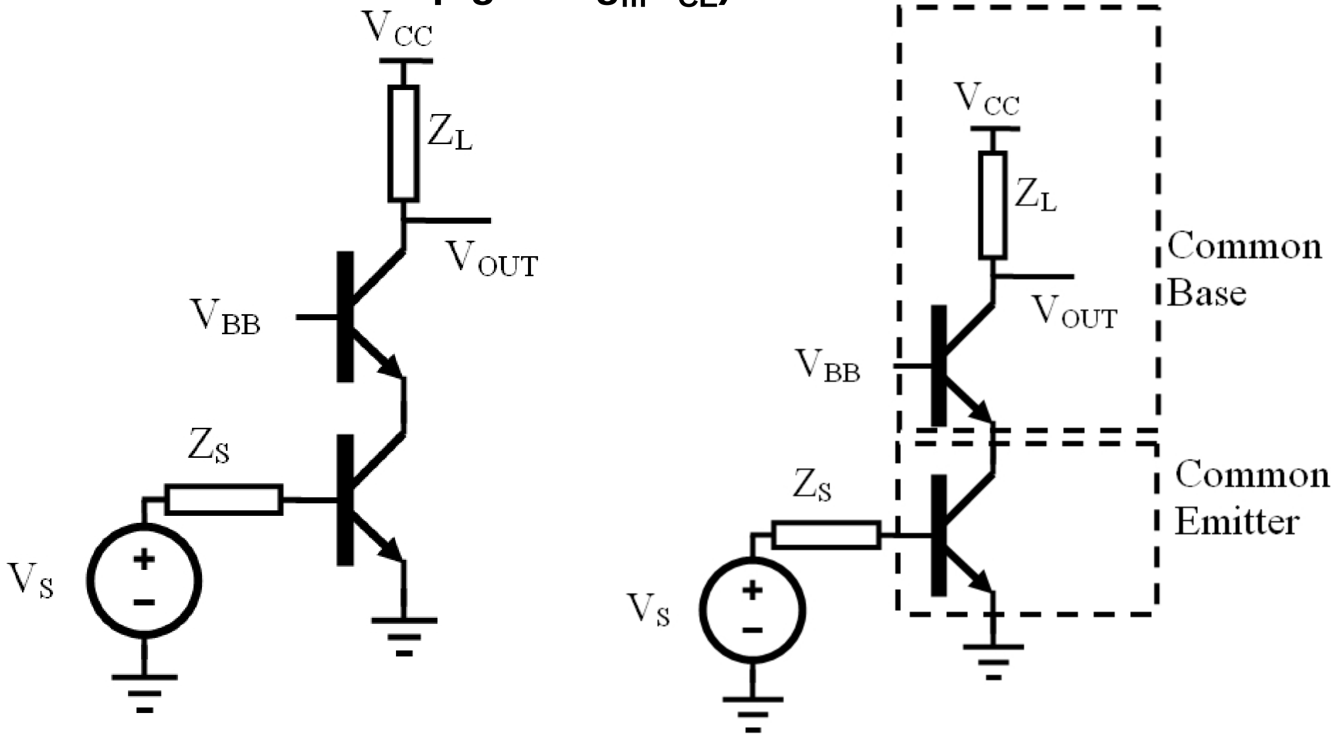
- Half circuit (i.e. driven from one side) is CC followed by CB
- Very wide frequency response
- Extremely high voltage gain





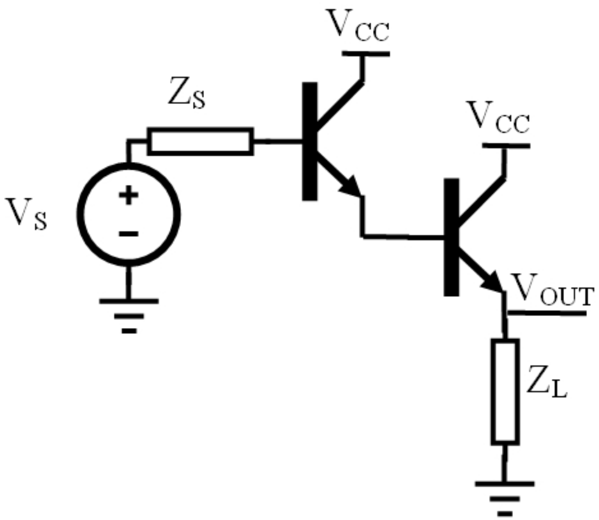
Cascode amplifier

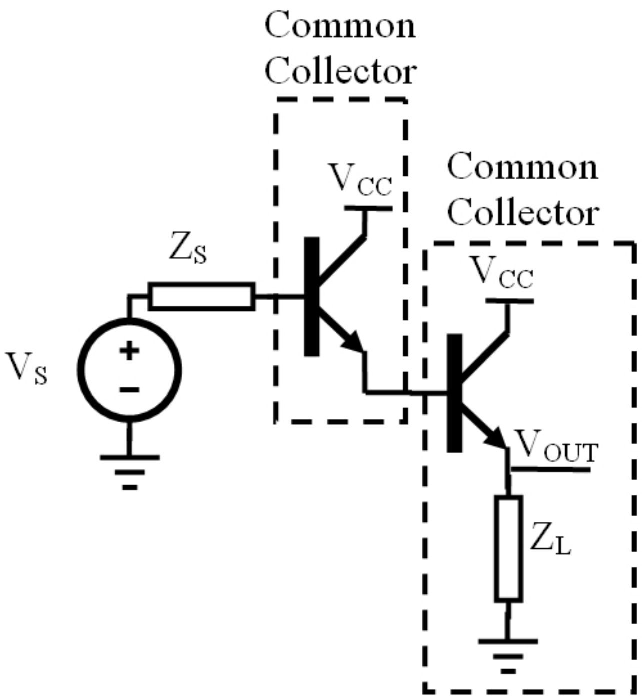
- **Wideband voltage amplifier**
- **CE stage operates at gain=-1, minimising miller loading of input.**
- **CB gives all the voltage gain, acting as transimpedance of value Z_L**
- **The cascode has a much higher output impedance (other than Z_L) than the CE amplifier (the common emitter Early resistance acts as series-series feedback to the common base with loop gain =$g_m R_{CE}$)**



Darlington pair

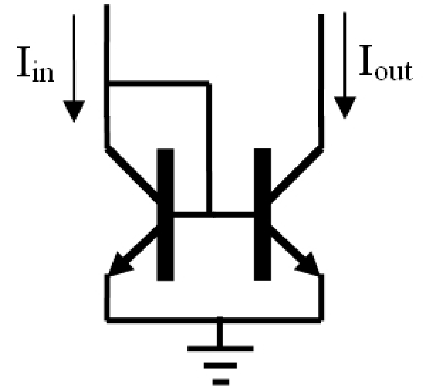
- **The darlington pair is a high gain power amplifier it has:**
  - ***Unity voltage gain***
  - ***High current gain equal to the product of the two transistor current gains***
- **Often used as a single transistor for higher beta. But :**
- **has high input DC voltage drop**
- **Good frequency response due to the absence of shunt Miller feedback.**
- **However, series Miller feedback introduces**
  driving capacitive loads.





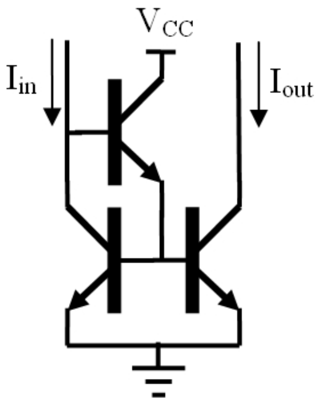
Current mirrors

- Use one transistor with unity feedback as a transimpedance amplifier to measure the V_{BE} required for a given current.
- Use a second transistor as transconductor to create a copy of the input current
- Can make a current amplifier by using larger output transistor.
- Current gain is in error due to base currents (i.e finite current gain)
- No DC gain error in FET mirrors (remember the AC current gain of a FET scales as the inverse of frequency!)
- Main source of error transistor mismatch
  - *“V_{BE} mismatch at a constant current” (BJT)*
  - *V_T mismatch in FET*
- AC analysis as in CE amplifier with extra source admittance due to input transistor
- Current mirrors are used for DC biasing multi-stage amplifiers
- Current mirrors often used load to a differential amplifier to turn the differential amplifier into a differential transconductor.



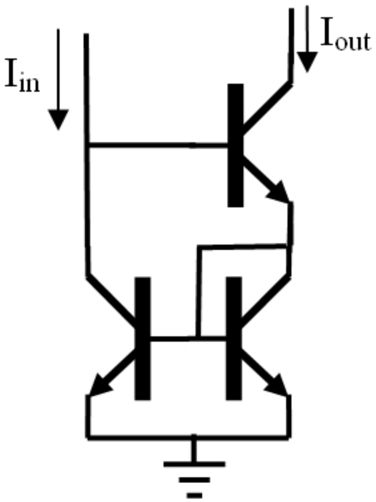
Simple current Mirror

Improved current mirrors



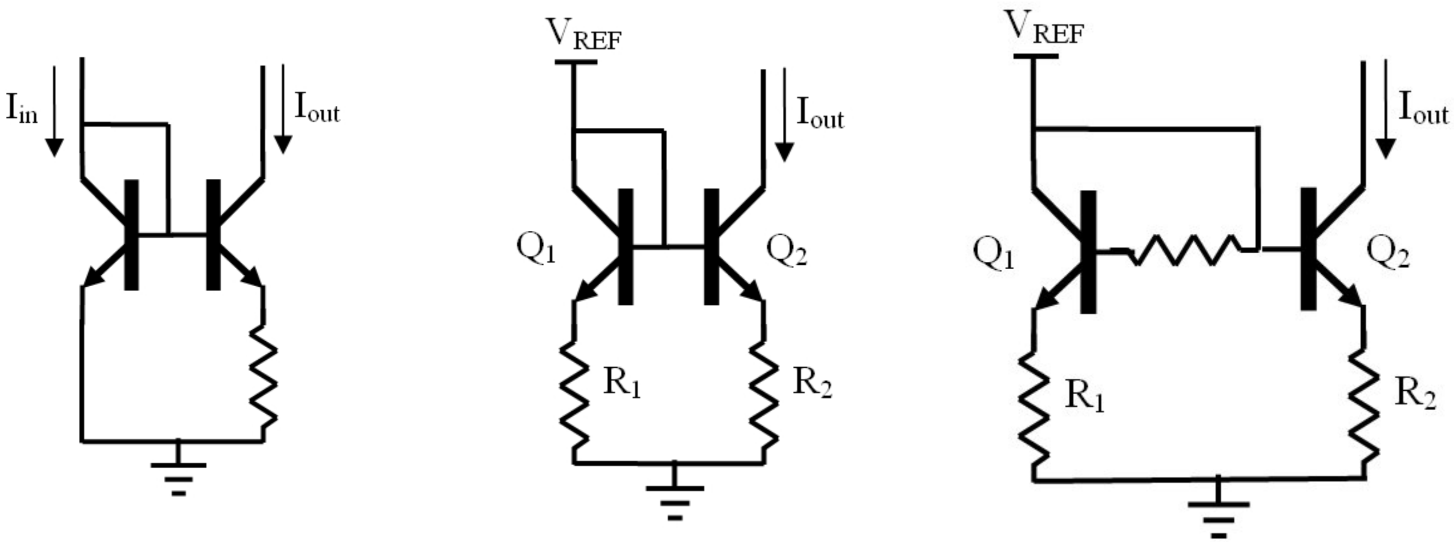
The buffered mirror

The CC amplifier feeding the bases reduces current gain error



The Wilson Mirror has high output Z, since output stage is a cascode amplifier

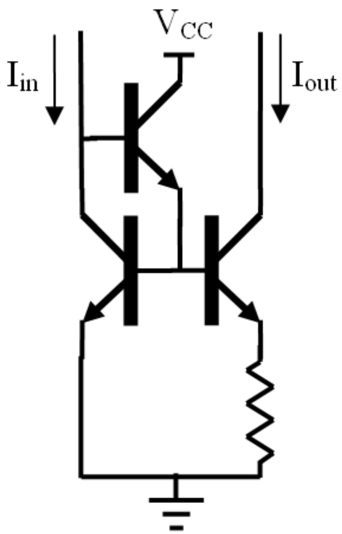
Scaling Mirrors: The Widlar mirror



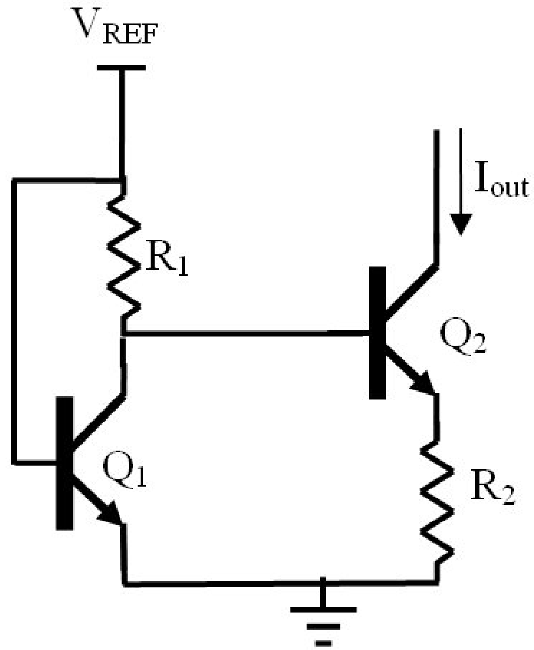
(a) ***(b)*** ***(c)***

- **The Widlar scaling mirror is often used as fixed scaling current source (a)**
- **Can be made as a buffered or a Wilson source (c)**
- **A feedback resistor can be added on the input side turning it into a transconductor (b)**
- **A base resistor as shown can provide “beta compensation” (i.e. introduce a zero in the frequency response (c)**

Some more mirrors



(a)

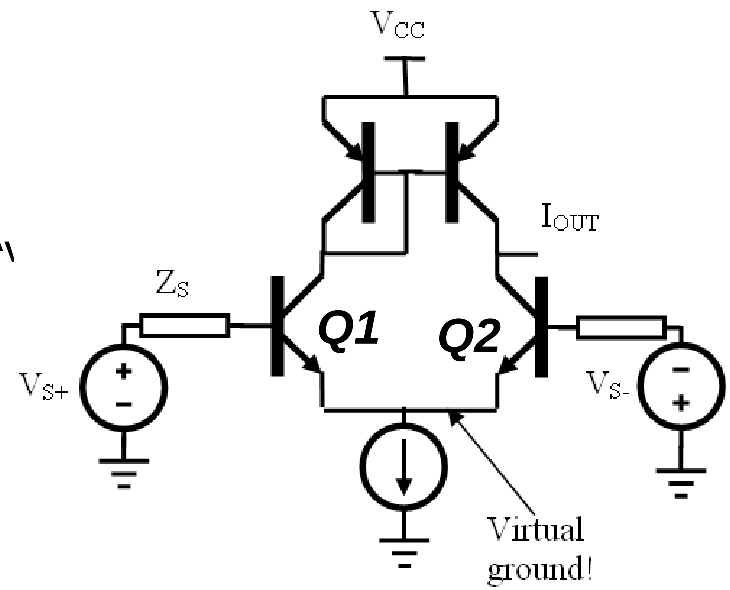


(b)

(a) Buffered Widlar mirror
(b) The "gm-compensated" mirror

Current mirror as a differential amp load

- The current mirror maps the left side current differential into the right side.
- The large signal response of this circuit is V_{out}=tanh(V_+-V_-)
- This circuit (a 3 stage amplifier! Why?)
- This circuit It has extremely high voltage gain: A_V of the order of V_A/V_{th}
- This circuit is also used for mixers if a transconductor is used in the place of the tail current source.
- There is no Miller effect on the left half circuit
- If this circuit drives a current sink at the output there is no Miller effect on the right half circuit either!
- The diff-amp has an extremely wide frequency response. This is partly a consequence of the resistive impedance match between the output of the first stage (emitter of Q1)and input of the second stage (emitter of Q2).



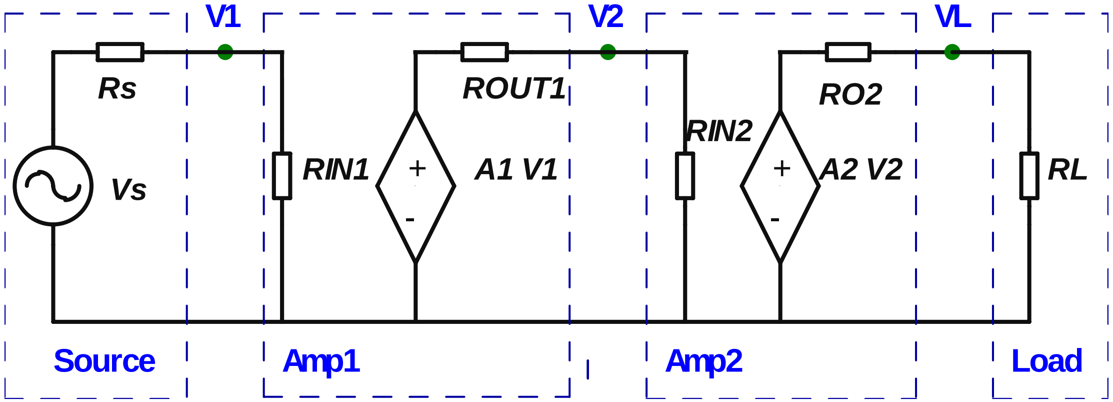
Two stage FET amplifiers

- ***The analogy we observed between single stage BJT and FET amplifiers applies, to two stage amplifiers. The correspondence is, as before, E→S, B→G, C→D.***
- ***The behaviour of BJT and FET configurations is very similar, except for the difference on the input side of the small signal equivalent circuit.***
- ***A very useful possibility opens up: Use a FET for one stage and a BJT for the other. Mixed bipolar-FET two-stage combinations try to exploit the smaller input admittance of FETs and the better frequency response and power handling capability of bipolars at the same time.***
- ***This approach gives rise to the "BiCMOS" manufacturing technologies which use FETs for input stages and BJTs for output stages, especially line drivers.***

	FET		Comments
Name	1st Stg	2nd Stg	
(voltage amp)	CS	CS	High Voltage gain
cascode	CS	CG	High bandwidth
(op-amp)	CS	CD	High Z_{in} low Z_{out}
(current buffer)	CG	CS	Higher Z_{out} than CB/CG
(current buffer)	CG	CG	Second stage to improve on CB/CG
(Not common)	CG	CD	Not common
(Not common)	CD	CS	Instead of CE, offers higher Z_{in}
differential amp	CD	CG	High voltage gain and bandwidth
darlington	CD	CD	High current gain

Multistage amplifiers

• Multistage amplifiers are difficult to compute if the components are not unilateral.
• For unilateral amplifiers things are simple. We multiply gains with appropriate voltage dividers.



$$\frac{V_L}{V_s} = \frac{1}{1+R_sY_{in1}} A_1 \frac{1}{1+R_{out1}Y_{in2}} A_2 \frac{1}{1+R_{o2}Y_L} \;, Y_x = \frac{1}{R_x} \;, x \in \{in1, in2, L\}$$

• For non-unilateral amplifiers:
  • The input impedance of each stage depends on the input impedance of the next stage
  • The output impedance of each stage depends on the output impedance of the preceding stage.
• This problem has a solution but involves the solution of sets of simultaneous quadratic equations.

Input - output impedance of a loaded amplifier

- **We calculate the input impedance of a voltage amplifier driving a load Z_L :**

$$\left.\begin{aligned} i_1 &= g_{11}v_1 + g_{12}i_2 \\ v_2 &= g_{21}v_1 + g_{22}i_2 \\ i_2 &= -v_2 Y_L \end{aligned}\right\} \Rightarrow \left.\begin{aligned} i_1 &= g_{11}v_1 - g_{12}Y_L v_2 \\ v_2 &= g_{21}v_1 - g_{22}Y_L v_2 \end{aligned}\right\} \Rightarrow$$

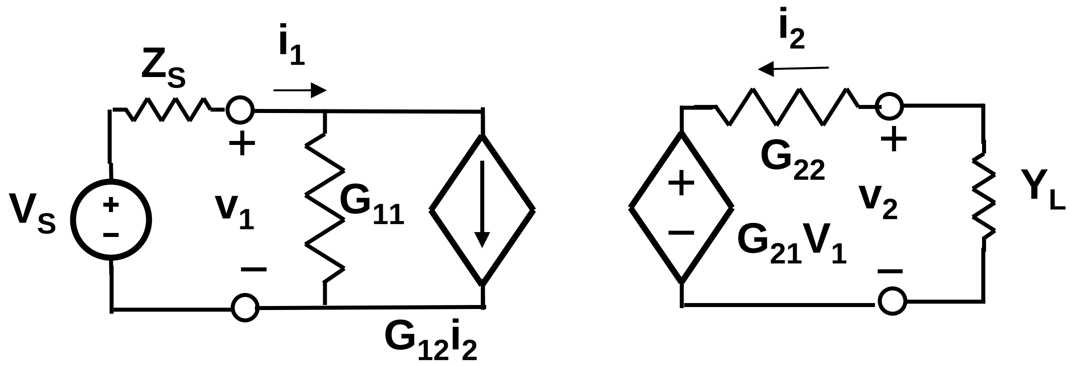
$$\left.\begin{aligned} g_{12}v_2 Y_L &= g_{11}v_1 - i_1 \\ \left(1 + g_{22}Y_L\right)v_2 &= g_{21}v_1 \end{aligned}\right\} \Rightarrow Z_{in} = \frac{v_1}{i_1} = \frac{1 + g_{22}Y_L}{g_{11} + \Delta_g Y_L}$$

- **A similar calculation for the output impedance of a voltage amplifier driven by a finite impedance Thevenin source Z_S gives:**

$$\left.\begin{aligned} i_1 &= g_{11}v_1 + g_{12}i_2 \\ v_2 &= g_{21}v_1 + g_{22}i_2 \\ v_1 &= v_s - i_1 Z_S \end{aligned}\right\} \Rightarrow \left.\begin{aligned} i_1 &= g_{11}\left(v_s - i_1 Z_S\right) + g_{12}i_2 \\ v_2 &= g_{21}\left(v_s - i_1 Z_S\right) + g_{22}i_2 \end{aligned}\right\} \Rightarrow$$

$$Z_{out} = dv_2 / di_2 = \frac{g_{22} + \Delta_g Z_S}{1 + g_{11}Z_S}$$

Gain of a fully loaded voltage amplifier



We start with the amplifier definition, plus the source-load boundary conditions:

$$i_1 = g_{11}v_1 + g_{12}i_2$$
$$v_2 = g_{21}v_1 + g_{22}i_2$$
$$v_1 = v_s - i_1 Z_s$$
$$i_2 = -v_2 Y_L$$

After some algebra we conclude that:

$$\frac{v_2}{v_s} = \frac{g_{21}}{(1+g_{11}Z_s)(1+g_{22}Y_L) - g_{12}g_{21}Z_sY_L} = \frac{g_{21}}{1+g_{11}Z_S + g_{22}Y_L + \Delta_g Y_L Z_s}, \quad \Delta_g = g_{11}g_{22} - g_{21}g_{12}$$

Cascade connection: Transmission Parameters

In a cascade connection,

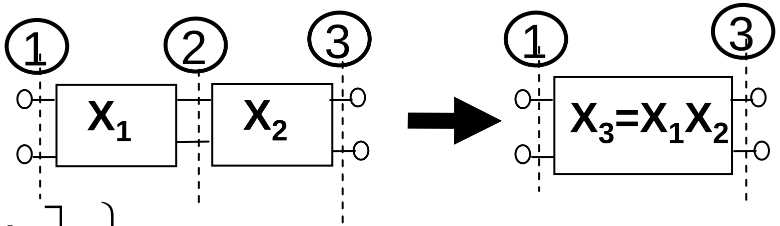
- V_1 of network X_2= V_2 of network X_1
- I_1 of network X_2 = -I_2 of network X_1

We can define a new set of parameters so that we have a simple way to calculate the response of cascades of amplifiers.

A suitable definition is:

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

With this definition, the ABCD parameters of a cascade of two networks are found from the matrix product of the individual ABCD matrices ports labelled for clarity):



$$\left.\begin{aligned} \begin{bmatrix} v_1 \\ i_1 \end{bmatrix} &= \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix} \\ \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix} &= \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} v_3 \\ -i_3 \end{bmatrix} \end{aligned}\right\} \Rightarrow \begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} v_3 \\ -i_3 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} \begin{bmatrix} v_3 \\ -i_3 \end{bmatrix}$$

Transmission (or ABCD) parameters (2)

The transmission matrix elements are related to the 4 gains:

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

$$A = \left.\frac{\partial v_1}{\partial v_2}\right|_{i_2=0} = \frac{1}{G_{21}} \qquad \text{B} = \left.\frac{-\partial v_1}{\partial i_2}\right|_{v_2=0} = \frac{-1}{Y_{21}}$$

$$C = \left.\frac{\partial i_1}{\partial v_2}\right|_{i_2=0} = \frac{1}{Z_{21}} \qquad \text{D} = \left.\frac{-\partial i_1}{\partial i_2}\right|_{v_2=0} = \frac{-1}{H_{21}}$$

A cascade of 2 amplifiers has gains:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} = \begin{bmatrix} A_1A_2 + B_1C_2 & A_1B_2 + B_1D_2 \\ C_1A_2 + D_1C_2 & C_1B_2 + D_1D_2 \end{bmatrix} \Rightarrow$$

$$g_f = \frac{1}{\dfrac{1}{g_{f1}g_{f2}} - \dfrac{1}{y_{f1}z_{f2}}} = \frac{g_{f1}g_{f2}y_{f1}z_{f2}}{y_{f1}z_{f2} - g_{f1}g_{f2}} \qquad y_f = \frac{1}{\dfrac{1}{g_{f1}y_{f2}} - \dfrac{1}{y_{f1}h_{f2}}} = \frac{g_{f1}y_{f2}y_{f1}h_{f2}}{y_{f1}h_{f2} - g_{f1}y_{f2}}$$

$$z_f = \frac{1}{\dfrac{1}{z_{f1}g_{f2}} - \dfrac{1}{h_{f1}z_{f2}}} = \frac{z_{f1}g_{f2}h_{f1}z_{f2}}{h_{f1}z_{f2} - z_{f1}g_{f2}} \qquad h_f = \frac{1}{\dfrac{1}{h_{f1}h_{f2}} - \dfrac{1}{z_{f1}y_{f2}}} = \frac{h_{f1}h_{f2}z_{f1}y_{f2}}{z_{f1}y_{f2} - h_{f1}h_{f2}}$$

Transmission (or ABCD) parameters (3)

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

- ***Note the sign of i_2 and also the reverse sense of signal flow. The sign is chosen so the ABCD matrix of a cascade of two networks is just the matrix product of the individual ABCD matrices (compare this to the messy loading calculation before!)***
- ***The reverse sense of signal flow is to keep the matrix finite if an amplifier is unilateral.***
- ***The conversion from, say, Y to ABCD follows the same logic as the Y(H) calculation:***

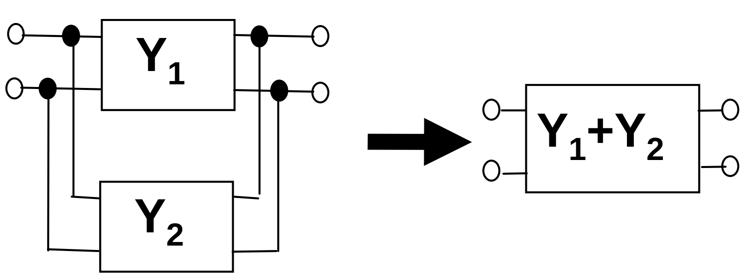
$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ Y_{11} & Y_{12} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -Y_{21} & -Y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{-1}{Y_{21}} \begin{bmatrix} Y_{22} & 1 \\ \Delta_Y & Y_{11} \end{bmatrix}, \ \Delta_y = Y_{11}Y_{22} - Y_{21}Y_{12}$$

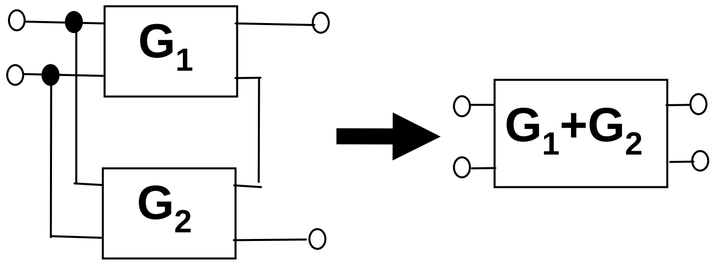
Remember that all ABCD parameters are inversely proportional to the gains. This is the reason for formally choosing port 2 as the input port.
The intuitive choice of input at port 1 would make all parameters inversely proportional to the reverse gains, which are small, and usually not very accurately determined.

Composition rules summary

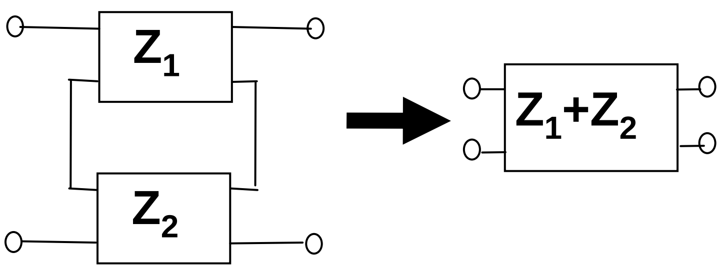
For the exact calculation of circuit interconnections we can use 2-port matrix algebra:



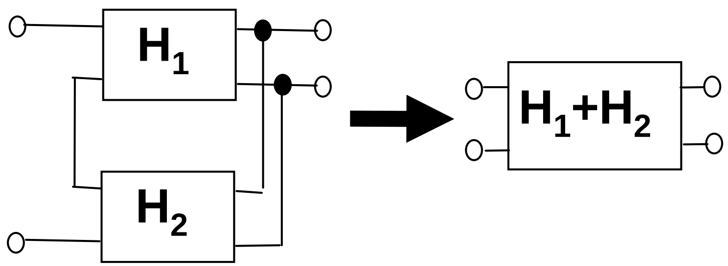
shunt-shunt: add Y matrices



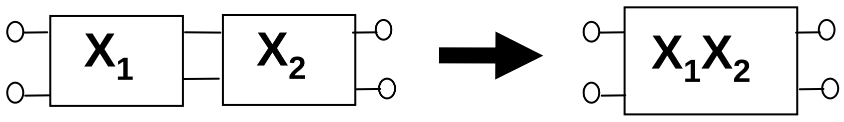
shunt-series: add G matrices



series-series: add Z matrices



series-shunt: add H matrices



cascade connection: multiply ABCD matrices

Multistage amplifiers: summary

- *Calculation of the response of unilateral multi-stage amplifiers is simple:*
  - *Product of gains and voltage dividers.*
- *Calculation of the response of non-unilateral multistage amplifiers involves determining for each stage:*
  - *the effect of source impedance on gain and output impedance*
  - *the effect of load impedance of input impedance and gain*
- *This typically leads to a set of simultaneous quadratic equations.*
- *A conceptually simpler analysis method involves the transmission or "ABCD" parameters which allow to describe all the loading effects of a non-unilateral cascade through a matrix product.*
- *With the introduction of ABCD parameters we have introduced simple ways to describe any connection between 2-port circuits.*