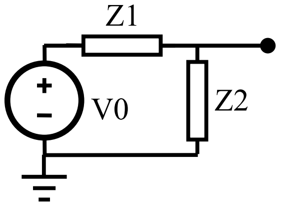
E2.2 Analogue electronics
Problem sheet 1 - ANSWERS

Q1. Calculate the Thevenin and Norton equivalent circuits of the voltage divider circuit for all the component combinations shown in the table:

	V0	Z1	Z2
	DC	Resistor	Resistor
	AC	Resistor	Capacitor
		Capacitor	Resistor
		Resistor	Inductor
		Inductor	Capacitor

ANSWER: Turn source off to find: $Z_T = \frac{1}{Y_N} = Z_1 // Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2}$

The open circuit voltage is the Thevenin voltage: $V_T = V_0 \frac{Z_2}{Z_1 + Z_2}$

The Norton current is the short circuit current, i.e.: $I_N = \frac{V_T}{Z_T} = \frac{V_0}{Z_1}$

The cases given:

V0	Z1	Z2	V_T	$Z_T = 1/Y_N$	I_N
DC	Resistor	Resistor	$\frac{V_0 Z_2}{Z_1 + Z_2}$	$\frac{R_1 R_2}{R_1 + R_2}$	$\frac{V_0}{Z_1}$
AC	Resistor	Capacitor	$\frac{V_0}{1 + j\omega R_1 C}$	$\frac{R_1}{1 + j\omega R_1 C}$	$\frac{V_0}{R_1}$
	Capacitor	Resistor	$\frac{j\omega C R_2 V_0}{1 + j\omega R_2 C}$	$\frac{R_2}{1 + j\omega R_2 C}$	$V_0 j\omega C$
	Resistor	Inductor	$\frac{V_0 j\omega L}{R_1 + j\omega L}$	$\frac{j\omega L R_1}{R_1 + j\omega L}$	$\frac{V_0}{R_1}$
	Inductor	Capacitor	$\frac{V_0}{1 - \omega^2 LC}$	$\frac{j\omega L}{1 - \omega^2 LC}$	$\frac{V_0}{j\omega L}$

Q2. Repeat Q1 but with a current source connected in the place of the voltage source.

ANSWER: The impedance Z1 is connected in series to a current source, so it is irrelevant. The Norton equivalent is just the source with Z2.

Q3. Calculate the small signal and the large signal Thevenin and Norton Equivalent circuits of a diode biased with a DC current of 1mA. The saturation current is 1fA. The thermal voltage is 25mV (at 17C)

ANSWER:

Large signal model:

The voltage developed on the diode with 1mA flowing is:

$$V_D = \frac{kT}{q}\ln\left(\frac{I_D}{I_0}\right) = 25mV \bullet \ln\left(10^{12}\right) = 25mV \bullet 27.6 \simeq 0.69V$$

This is also the large signal Thevenin voltage.

The short circuit current is clearly 1mA.

So the large signal equivalent is:

$V_T = 0.69V$, $\mathrm{I}_N = 1mA$, $Z_T = 690\Omega$

The small signal equivalent is again $V_T = V_{OC} = 0.69V$

The small signal Thevenin resistance is quite different than the large signal Thevenin resistance:

$$Y_N = \frac{\partial I_D}{\partial V_D} = \frac{\partial\left(I_0 e^{qV_D/kT}\right)}{\partial V_D} = \frac{I_D}{V_T} = \frac{1mA}{25mV} = 40\Omega$$

The Norton current has to be (for consistency!)

$$I_N = \frac{V_T}{Z_T} = \frac{0.69}{40} = 17.25mA$$

This is the case where the small and large signal Thevenin models are quite different!

Q4. A series connection of a diode and a 1kΩ resistor embedded in a big circuit develops 1V DC across it. Calculate the small signal Thevenin resistance of the resistor-diode connection. The diode saturation current is 1fA. The thermal voltage is 25mV (at 17C) (Do not include the voltage source in the calculation, only the diode and the resistor!)

ANSWER

We need to find the voltage and current developed on the diode. So we need to solve:

$$\underbrace{I_D R}_{A} + \underbrace{\frac{kT}{q}\ln\left(\frac{I_D}{I_0}\right)}_{B} = 1V$$

This can be done, for example, by iteration:

We assign the voltage B a starting value, eg B=0.5V then we compute A=0.5V and $I = V_A / 1k\Omega$

We can then compute a new value for the B term:

$V_B = \frac{kT}{q}\ln\left(\frac{I}{I_0}\right)$ and repeat until the two voltages converge

This is done below:

Iteration	A	B	I
1	0.5	0.5	0.0005
2	0.326553	0.673447	0.000327
3	0.337204	0.662796	0.000337
4	0.336401	0.663599	0.000336
5	0.336461	0.663539	0.000336

We can now calculate the small signal Thevenin impedance:

$$Z_T = 1k\Omega + \frac{25mV}{0.336mA} = 3972\Omega$$

Q5. In the circuit diagram above, assume both Z1 and Z2 are arbitrary complex impedances and V0 a sinusoidal source of amplitude V0 and at a frequency ω.

1. Derive an expression for the average power PZ2 dissipated in Z2.
2. Derive also an expression for the average total power PT delivered by the source V0 (which is the power dissipated in Z1 and Z2)
3. What is the power delivered to Z2 if Z1 is finite and Z2 is zero?
4. What is the power delivered to Z2 if Z1 is finite and Z2 is infinite?
5. For what value of Z2 is PZ2 maximum, if Z1 is given? What is the maximum fraction of PT that can be delivered to the load?

ANSWER:

a) $\langle P_{Z2}\rangle = \langle \operatorname{Re} I V_{Z2}^{*}\rangle = \langle |I|^2\rangle \operatorname{Re} Z_2 = \langle |V|^2\rangle \dfrac{R_2}{|Z_1+Z_2|^2} = \langle |V|^2\rangle \dfrac{R_2}{(R_1+R_2)^2+(X_1+X_2)^2}$

b) $\langle P_T\rangle = \langle \operatorname{Re} I V^{*}\rangle = \langle |I|^2\rangle \operatorname{Re}(Z_1+Z_2) = \langle |V|^2\rangle \dfrac{R_1+R_2}{|Z_1+Z_2|^2} = \langle |V|^2\rangle \dfrac{R_1+R_2}{(R_1+R_2)^2+(X_1+X_2)^2}$

c) zero (VZ2=0)
d) zero (IZ2=0)
e)

maximum of $\langle P_{Z2}\rangle = \langle |V|^2\rangle \dfrac{R_2}{(R_1+R_2)^2+(X_1+X_2)^2}$ occurs if $X_2 = -X_1$

Then the maximum of

$\langle P_{Z2}\rangle = \langle |V|^2\rangle \dfrac{R_2}{(R_1+R_2)^2}$ occurs for

$$\frac{\partial}{\partial R_2}\langle P_{Z2}\rangle = 0 \Rightarrow \langle |V|^2\rangle \frac{\partial}{\partial R_2}\frac{R_2}{(R_1+R_2)^2} = 0 \Rightarrow \langle |V|^2\rangle \frac{R_1-R_2}{(R_1+R_2)^3} = 0 \Rightarrow R_1 = R_2$$

This is ½ of PT.