E2.2 Analogue electronics
Problem sheet 2 (Week 4)

Q1: Determine the following small signal model parameters of an NPN bipolar transistor biased connected as a Common Emitter amplifier and biased with a DC collector current of 1mA and 10mA. The transistor DC current gain is $\beta = 200$ and the Early voltage is $V_A = 100V$:

- Transconductance g_m
- Hybrid Pi input resistance R_π
- Collector output resistance R_{CE}

Answer:

$$g_m = \frac{I_C}{V_{th}} \Rightarrow \begin{cases} @\, I_C = 1mA\ ,\ g_m = \dfrac{1mA}{20mV} = 40mS \\ @\, I_C = 10mA\ ,\ g_m = \dfrac{10mA}{20mV} = 400mS \end{cases}$$

$$R_\pi = \frac{\beta}{g_m} \Rightarrow \begin{cases} @\, I_C = 1mA\ ,\ R_\pi = \dfrac{200}{40mS} = 5\ \text{k}\Omega \\ @\, I_C = 10mA\ ,\ R_\pi = \dfrac{200}{400mS} = 500\ \Omega \end{cases}$$

$$R_{CE} = \frac{V_A}{I_C} \Rightarrow \begin{cases} @\, I_C = 1mA\ ,\ R_{CE} = \dfrac{100\ \text{V}}{1mA} = 100\ \text{k}\Omega \\ @\, I_C = 10mA\ ,\ R_{CE} = \dfrac{100\ \text{V}}{10mA} = 10\ \text{k}\Omega \end{cases}$$

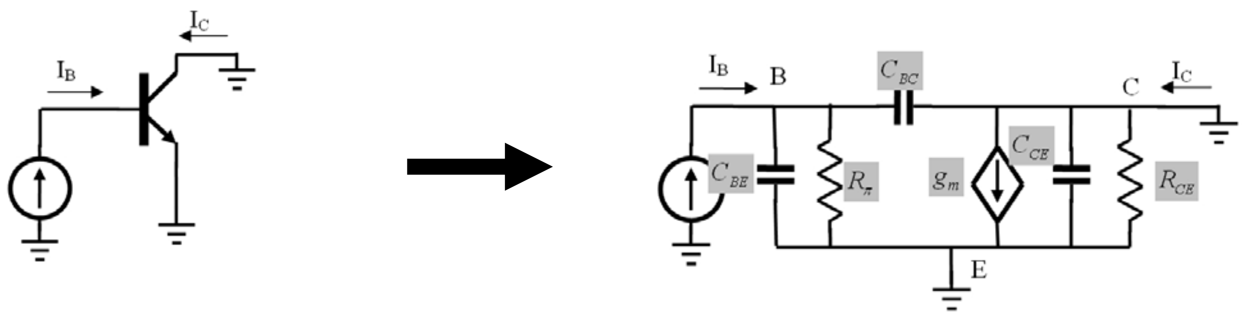
Q2: Repeat question 1 if the transistor is PNP but all its other specifications are the same with those of the NPN transistor in question 1.

Answer: Same answers, nothing depends on whether the transistor is NPN or PNP

Q3: Draw the small signal equivalent circuit for an NPN bipolar transistor, including capacitances. Label all components. Assume the transistor is properly biased, and is connected as a current amplifier: The emitter is grounded, the base is driven by a current source and the collector is connected to the power supply through a current meter.

Calculate the small signal current gain of this amplifier as a function of frequency. Express this gain as a function of the equivalent circuit elements. Calculate the current gain at low and at high frequencies. At what frequency does the current gain start being frequency dependent?

Answer:



$$\left.\begin{aligned} i_B &= \frac{V_{BE}}{\dfrac{R_\pi}{1+j\omega R_\pi\left(C_{BE}+C_{CE}\right)}} = \frac{g_m V_{BE}\left(1+j\omega R_\pi\left(C_{BE}+C_{CE}\right)\right)}{\beta_0} \\ i_C &= g_m V_{BE} \end{aligned}\right\} \Rightarrow$$

$$\beta(\omega) = \frac{i_C}{i_B} = \frac{\beta_0 g_m V_{BE}}{g_m V_{BE}\left(1+j\omega R_\pi\left(C_{BE}+C_{CE}\right)\right)} = \frac{\beta_0}{1+j\omega R_\pi\left(C_{BE}+C_{CE}\right)}$$

At low frequencies the current gain is essentially constant and equal to β_0. At high frequencies the dependence is:

$$\beta(\omega) \simeq \frac{\beta_0}{j\omega R_\pi\left(C_{BE}+C_{CE}\right)} = \frac{g_m}{j\omega\left(C_{BE}+C_{CE}\right)} = \frac{-j\omega_T}{\omega}\ ,\ \omega_T = \frac{g_m}{C_{BE}+C_{CE}}$$

The transition between the two behaviours happens at

$$\omega R_\pi\left(C_{BE}+C_{CE}\right) = 1 \Rightarrow \omega = \frac{1}{R_\pi\left(C_{BE}+C_{CE}\right)} = \frac{g_m}{\beta_0\left(C_{BE}+C_{CE}\right)} = \frac{\omega_T}{\beta_0}$$

Q4: Determine the C_{BE} and C_{BC} of an NPN bipolar transistor which has an $f_T = 1$ GHz at a bias current $I_C = 1mA$ if $C_{BE} = 9C_{BC}$.

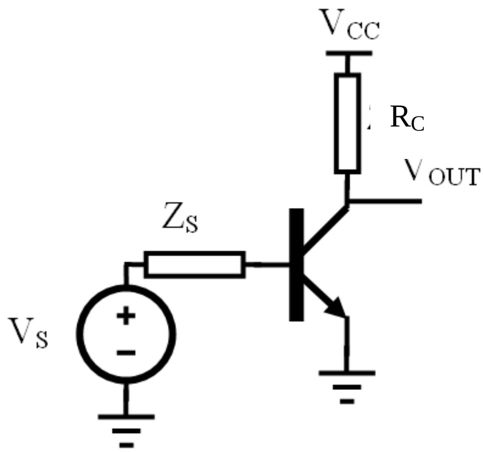
Answer:

$$\omega_T = \frac{g_m}{C_{BE}+C_{CE}} = \frac{40mS}{10C_{CE}} = 2\pi\times 10^9 \Rightarrow C_{CE} = 636\,fF \Rightarrow C_{BE} = 5.73\,pF$$

Q5: Draw a diagram of an NPN bipolar transistor connected as a common Emitter voltage amplifier. The base is driven by an ideal voltage source which includes a DC bias V_0 so that the DC collector current is 1mA, and a small AC component v so that $V_{BE} = V_0 + v$.

- Choose a load resistance R_C so that the DC gain is 200
- Choose a DC Collector supply so that the collector has a symmetric maximum swing. The Saturation voltage is 0.2V

Answer:

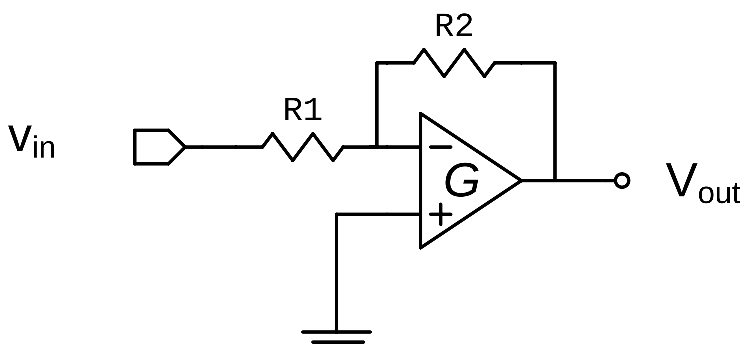


The DC gain is $A_v = -g_m R_C \Rightarrow R_C = 200/40mS = 5\ \text{k}\Omega$

The DC drop on R_C is 5V, so the supply needs to be $V_{CC} = V_{SAT} + 2V_{RC} = 10.2V$

Q6: Use the Miller Theorem to calculate symbolically the input impedance of an inverting amplifier built with an op-amp whose open loop gain is G=10.

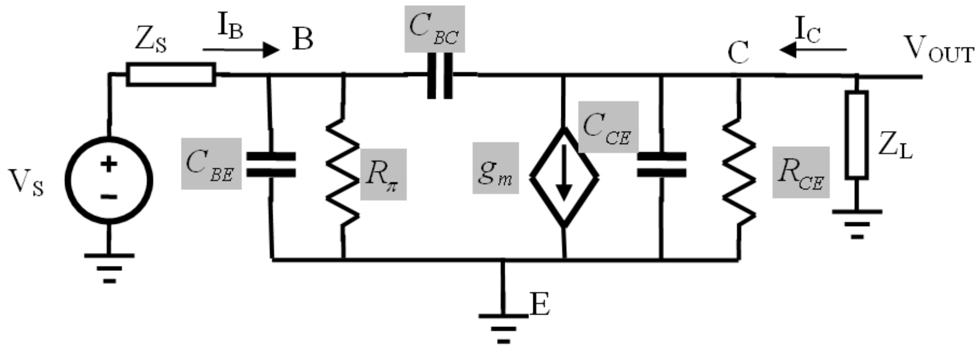
Answer:

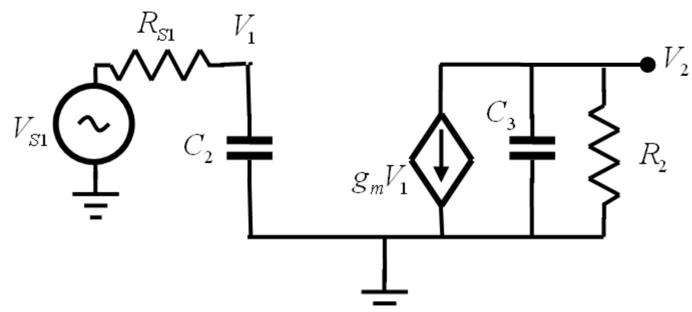


Since the op-amp gain is G=10, it follows that the input impedance is
$Z_{in} = R_1 + R_2/(G+1) = R_1 + R_2/11$

Q7: Draw a small signal model of the circuit in Question 5. Use the Miller Theorem to simplify the circuit. Assume that for this circuit the results of Question 4 are valid. Calculate the frequency response of the voltage gain of this amplifier in the following 2 cases:

- The signal source has a Thevenin impedance $R_T = 0$
- The signal source has a Thevenin impedance $R_T = 1\ \text{k}\Omega$





$$R_{S1} = R_\pi // R_S = \frac{Z_S}{1 + Z_S / R_\pi}$$

$$C_2 = C_{BE} + \left(-A_V + 1\right) C_{BC} = 9C_{BC} + 201C_{BC} = 210C_{BC} = 1.2nF$$

$$R_2 = R_{CE} // R_L = \frac{R_L}{1 + R_L / R_{CE}}$$

$$V_{S_1} = \frac{V_S}{1 + Z_S / R_\pi}$$

If $R_S = 0$ then $A_v\left(\omega\right) = \frac{v_c}{v_s} = -g_m R_2 \frac{1}{1 + j\omega R_2 C_3}$

If R_S is finite, then

$$A_v\left(\omega\right) = \frac{v_c}{v_s} = -\frac{g_m v_1 R_2}{v_s} = -\frac{-g_m R_2}{1 + Z_S / R_\pi} \frac{1}{1 + R_{S1} j\omega C_2} \frac{1}{1 + j\omega R_2 C_3}$$