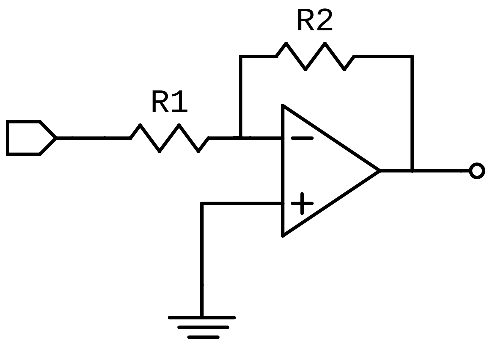
E2.2 Analogue electronics
Problem sheet 3 (Week 5)

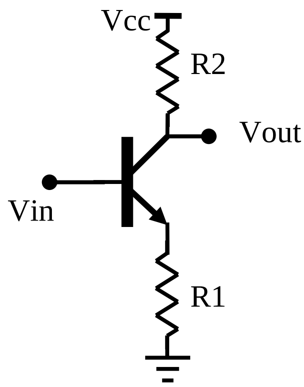
Q1: **(Tutorial question for Week 6/7)**

a) What is the difference between an ideal op-amp, an ideal voltage amplifier and a commercial op-amp?
b) Write an expression for the input impedance of the inverting amplifier below if R1=1k, R2=100k, the op-amp has a DC gain G=10^4 and its dominant pole is at f=1.59Hz. Evaluate the input impedance of this circuit at f=15.9 kHz. The op-amp has zero input conductance and output resistance.

The open loop gain of a "dominant pole amplifier" is given by $G(f) = \frac{G_{DC}}{1+s\tau} = \frac{G_{DC}}{1+s/\omega_0}$, with G_{DC} the DC gain and $\omega_0 = 1/\tau$ the pole (also called the "break") frequency.

c) Calculate the DC voltage gain of the emitter degenerated common emitter amplifier below if R1=50 Ohms, R2=200 ohms and I_C=1mA. (leave this item for week 6)





ANSWER:

d) An ideal op-amp has infinite gain, zero input conductance, zero output resistance and zero reverse gain.
An ideal voltage amplifiers has **finite** gain , zero input conductance, zero output resistance and zero reverse gain.
A commercial op-amp has a large (but not infinite) gain, a small (but not zero) input conductance and output resistance. It also has negligibly small (but not zero) reverse current gain.

e) The op-amp gain is: $G(\omega) = \frac{G_0}{1+j\omega/\omega_0}$ with $G_0 = 10^4$ and $\omega_0 = 10$ rad/s . Using the Miller Theorem, the input impedance of this circuit is: $Z_{in} = R_1 + R_2 / \left(1 + \frac{G_0}{1+j\omega/\omega_0}\right)$.

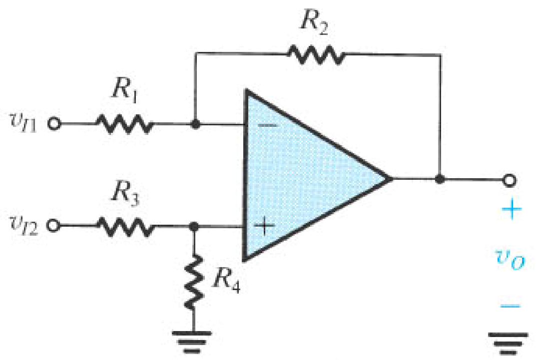
$\omega_0 = 10$ rad/s .At f=15.9 krad/s, $\omega_0 = 10^5$ rad/s and

$$Z_{in} = 1\ \text{k}\Omega + 100\ \text{k}\Omega / \left(1 + \frac{10^4}{1+j10^5/10^5}\right) = 1\ \text{k}\Omega + 100\ \text{k}\Omega / (1-j)$$

f) The gain of the emitter degenerated amplifier is $G = -g_m{}' R_C$ where $g_m{}'$ is the closed loop transconductance: $g_m{}' = \frac{-g_m}{1+g_m R_E}$. At a 1 mA DC bias, and the resistor values given, $g_m = 40 \text{ mS} \Rightarrow g_m{}' = \frac{0.04}{1+0.04\times 50} = 0.0133 \Rightarrow G = -2.66$. Please note that this is not equal to the ratio of -R2/R1, a useful approximation when $g_m R_E >> 1$

Q2. Use superposition to show that the following circuit, built with an ideal op-amp, has an output equal to: $v_o = v_{i2}\left(\frac{1+R_2/R_1}{1+R_3/R_4}\right) - v_{i1}\frac{R_2}{R_1}$

Chose relative resistor values so that $v_{out} = A(v_{i1} - v_{i2})$. what is the value of the constant A?



ANSWER:

$$v_+ = \frac{R_4}{R_3+R_4} v_{i2} = v_- = v_{i1}\frac{R_2}{R_1+R_2} + v_o\frac{R_1}{R_1+R_2} \Rightarrow v_o = \frac{R_1+R_2}{R_1}\left(v_{i2}\frac{R_4}{R_3+R_4} - v_{i1}\frac{R_2}{R_1+R_2}\right)$$

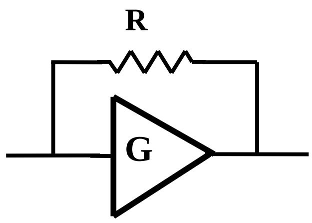
$$v_o = v_{i2}\frac{R_4(R_1+R_2)}{R_1(R_3+R_4)} - v_{i1}\frac{R_2}{R_1} \Rightarrow v_o = v_{i2}\left(\frac{1+R_2/R_1}{1+R_3/R_4}\right) - v_{i1}\frac{R_2}{R_1}$$

To prove we can get $v_{out} = A(v_{i1} - v_{i2})$ we need

$\frac{R_4(R_1+R_2)}{R_1(R_3+R_4)} = \frac{R_2}{R_1} \Rightarrow R_4 R_1 = R_2 R_3$. The gain is simply $A = \frac{R_2}{R_1}$

Q3: Apply the Miller Theorem to how that the input impedance of the following circuit is: $Z_{in} = \frac{R}{1-G}$. Derive an expression for the input impedance of this circuit if G is a dominant pole

inverting amplifier, the open loop gain of the amplifier is given by $G(f)=\frac{G_{DC}}{1+s\tau}=\frac{G_{DC}}{1+s/\omega_0}$.
What are the break frequencies of the impedance as a function of frequency? What is the input impedance at low frequencies and at very high frequencies?
The open loop gain of a "dominant pole amplifier" is given by $G(f)=\frac{G_{DC}}{1+s\tau}=\frac{G_{DC}}{1+s/\omega_0}$, with G_{DC} the DC gain and $\omega_0 = 1/\tau$ the pole (also called the "break") frequency.



ANSWER:

$v_{out} = Gv_{in}$ Ohm's law applies on R so that $i_R = \frac{v_{in}-v_{out}}{R} = \frac{(1-G)v_{in}}{R}$. This relation describes a resistor of value $Z_{in} = \frac{R}{1-G}$ as required. Alternatively, the answer is obtained by inspection, by applying the Miller theorem.

If G is an inverting dominant pole amplifier then,

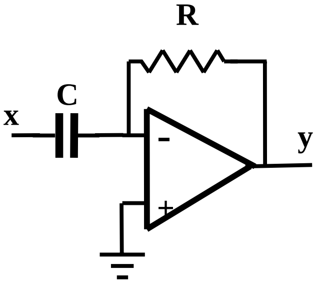
$$Z_{in} = \frac{R}{1+\frac{G_{DC}}{1+s\tau}} = R\frac{1+s\tau}{G_{DC}+1+s\tau} = \frac{R}{(G_{DC}+1)}\frac{1+s\tau}{\left(1+s\tau/(1+G_{DC})\right)}$$

This impedance function has a zero at $f_z = \frac{1}{2\pi\tau}$ and a pole at $f_p = \frac{G_{DC}+1}{2\pi\tau} = (G_{DC}+1)\, f_z$
At low frequencies the impedance goes to $R/(G_{DC}+1)$. At high frequencies it approaches R.

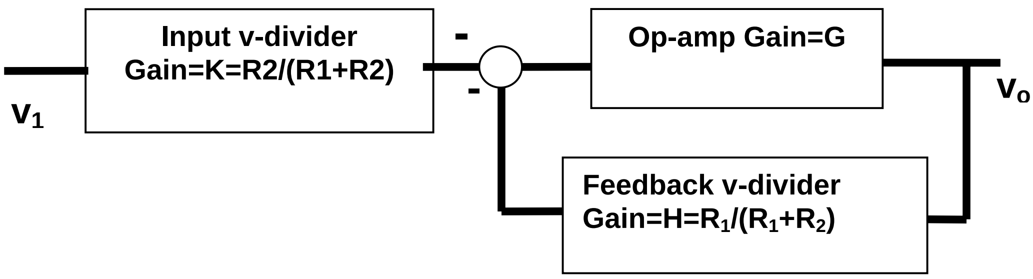
Q4: Show that the transfer function of the ideal differentiator below constructed with a dominant pole amplifier is:

$$H(s) = \frac{v_{out}}{v_{in}} = \cdots = \frac{-sRCG_{DC}}{s^2 \tau RC + s(\tau + RC) + (G_{DC} + 1)}$$

Calculate the maximum gain of this filter which occurs roughly when denominator is purely imaginary.



ANSWER:
The inverting amplifier is modelled as



The block gains are

$$K = \frac{R}{R + 1/sC} = \frac{s\tau_0}{s\tau_0 + 1} \quad \text{and} \quad H = \frac{1/sC}{R + 1/sC} = \frac{1}{s\tau_0 + 1}\ ,\ \tau_0 = RC$$

The overall closed loop gain is

$$A_v = \frac{-KG}{1+GH} = \frac{-\frac{s\tau_0}{s\tau_0+1}\frac{G_{DC}}{1+s\tau}}{1+\frac{1}{s\tau_0+1}\frac{G_{DC}}{1+s\tau}} = \frac{-s\tau_0 G_{DC}}{(s\tau+1)(s\tau_0+1)+G_{DC}} = \frac{-s\tau_0 G_{DC}}{s^2\tau\tau_0 + s(\tau+\tau_0) + (G_{DC}+1)} \quad \text{QED}$$

Since $G(f) = \frac{G_{DC}}{1+s\tau} = \frac{G_{DC}}{1+s/\omega_0}$

The maximum amplitude, which occurs near $s^2\tau\tau_0 + (G_{DC}+1) = 0 \Rightarrow \omega_{max} = \sqrt{\frac{G_{DC}+1}{\tau\tau_0}}$, is

$$|A_{v\max}| = \frac{-\omega_{max}\tau_0 G_{DC}}{\omega_{max}(\tau+\tau_0)} = \frac{-\tau_0 G_{DC}}{(\tau+\tau_0)}$$