E2.2 Analogue electronics
Problem sheet 4 (Week 7)

A voltage amplifier is given, which is to be used, with different kinds of feedback connections, to synthesise a number of different circuit building blocks. The amplifier has an input impedance $Z_{in} = 10k\Omega$, output impedance $Z_{out} = 100\Omega$ and an open loop gain $G = 100$. The gain is constant over a frequency range of interest. You may assume that the feedback network impedance has a negligible effect on input and output impedances of all the feedback amplifiers mentioned below.

Q1:

a) Derive an expression and a value for the current gain of this amplifier if it is connected like a current amplifier (between a current source and a current meter).
b) Derive an expression and a value for the maximum power gain of this amplifier. Express the maximum power gain in terms of the voltage and current gain.

HINT: Given that the maximum power is delivered to a matched load, you should load this amplifier with $R_L = Z_{out}$.

ANSWER

a) $I_{in} = \frac{V_{in}}{Z_{in}}, I_{out} = \frac{-V_{out}}{Z_{out}} \Rightarrow \beta = \frac{I_{out}}{I_{in}} = \frac{-GZ_{in}}{Z_{out}} = -10^4$

b) $P_{in} = \frac{V_{in}^2}{Z_{in}}, P_{out} = \frac{G^2 V_{in}^2}{4Z_{out}} \Rightarrow G_P = \frac{G^2 Z_{in}}{4Z_{out}} = \frac{G\beta}{4} = 2.5 \cdot 10^5$

Q2: You decide to use negative feedback to obtain a closed loop voltage gain $G_{CL} = 10$ while increasing the input impedance and reducing the output impedance.

a) What kind of negative feedback connection should you use?
b) What is the magnitude of the loop gain required to do this? What is the feedback path gain?
c) What is the closed loop input and output impedance of the feedback amplifier?

ANSWER

a) series shunt
b) $F = 1 + GH = 10 \Rightarrow GH = 9 \Rightarrow H = 0.09$
c) $Z_{in}' = Z_{in}(1 + GH) = 100k\Omega$
$Z_{out}' = Z_{out} / (1 + GH) = 10\Omega$

Q3: The same amplifier is used with negative feedback to make a current amplifier.

a) What kind of negative feedback connection should you use in order to reduce the input impedance and increase the output impedance at the same time?
b) What should the loop gain be in order to make the output impedance 10 times bigger than the input impedance? Is it possible to use this voltage amplifier to make a current amplifier with $Z_{out} = 1000 Z_{in}$?

c) The current gain of a voltage amplifier used as a current amplifier is (Q1): $\beta = \frac{-GZ_{in}}{Z_{out}}$.

What is the closed loop current gain of the feedback amplifier which has $Z_{out} = 10Z_{in}$?

ANSWER:

a) shunt-series

b) we require

$$Z'_{in} = Z_{in} / (1+GH) = 0.1 \cdot Z'_{out} = 0.1 \cdot Z_{out}(1+GH) \Rightarrow$$

$$10^4\Omega = 10\Omega(1+GH)^2 \Rightarrow (1+GH)^2 = 1000 \Rightarrow$$

$$1+GH = 31.6 \Rightarrow GH = 30.6$$

To make an amplifier with $Z_{out} = 1000Z_{in}$ we would need

$$Z'_{in} = Z_{in} / (1+GH) = 0.001 \cdot Z'_{out} = 0.001 \cdot Z_{out}(1+GH) \Rightarrow$$

$$10^4\Omega = 0.1\Omega \cdot (1+GH)^2 \Rightarrow (1+GH)^2 = 10^5 \Rightarrow 1+GH = 316 \Rightarrow GH = 315 > G$$

This is not possible unless an amplifier is used in the feedback path to get $H > 1$

c) The closed loop current gain will be:

$$\beta_{CL} = \frac{\beta_{OL}}{F} = \frac{-GZ_{in}}{Z_{out} \cdot 31.6} = 316$$

Q4: The same amplifier is used with **positive** feedback to make a current amplifier.

a) What kind of positive feedback connection should you use in order to reduce the input impedance and increase the output impedance at the same time?
b) What is the loop gain required to make the output impedance 1000 times bigger than the input impedance?
c) What is the closed loop current gain of the feedback amplifier described in this question? What are the values of Z_{in} and Z_{out} ?

ANSWER:

a) Series – Shunt

b)

$$Z'_{in} = Z_{in}(1-GH) = 10^{-3} \cdot Z'_{out} = 10^{-3} Z_{out} / (1-GH) \Rightarrow$$

$$10^{-3}\frac{Z_{out}}{Z_{in}} = (1-GH)^2 \Rightarrow (1-GH)^2 = 10^{-5} \Rightarrow 1-GH = 3.16 \cdot 10^{-3} \Rightarrow$$

$$GH = 0.997 \Rightarrow H = 3.16 \cdot 10^{-6}$$

c) the feedback factor is $3.16 \cdot 10^{-3}$ so the current gain is 300 times larger than the open loop gain. Therefore $\beta = 3.16 \cdot 10^6$.

d) Also, $Z'_{in} = Z_{in}(1-GH) = 31.6\Omega$ and $Z'_{out} = Z_{out} / (1-GH) = 31.6k\Omega$

Q5. The same amplifier is used with a suitable feedback connection to make a transconductance amplifier.

a) What type of negative feedback connection could you use to do this? Why?
b) What type of **positive** feedback connection could you use to get the same terminal impedances as in part Q4a?
c) Derive a relation between the feedback path gains for the negative and positive feedback connections described in parts a) and b) above.
d) Derive an expression for the ratio of the transconductance gains in parts a) and b) in terms of the negative feedback connection feedback factor. Which of the transconductances , achieved by positive and negative feedback, respectively, is larger?

ANSWER:

a) Need to increase input impedance, i.e. series input connection, and increase output impedance, i.e. again series connection at the output.
b) With positive feedback both connections need to be shunt connections.
c) $F_N = 1/F_p \Rightarrow 1 + GH_N = \frac{1}{1-GH_P} \Rightarrow H_N = \frac{1}{G}\left(\frac{1}{1-GH_P} - 1\right) = \frac{H_P}{1-GH_P}$
d) $g_{mp} = \frac{g_m}{F_N}, g_{mn} = \frac{g_m}{F_P} \Rightarrow \frac{g_{mn}}{g_{mp}} = \frac{F_P}{F_N} = F_N^{-2}$. Since $F_N = 1/F_p$.

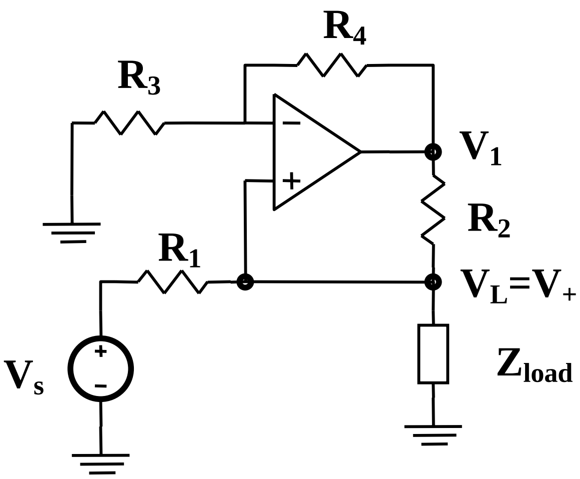
Given that $F_n > 1, F_p < 1$ it follows that $g_{mn} < g_{mp}$

Q6: Prove that the following circuit behaves like a current source when $R_1 R_4 = R_2 R_3$ and the op-amp is considered to be ideal. Compare the positive and negative feedback path gains.

a) Show that the positive feedback path gain is reduced when we connect a load of finite admittance to the output.
b) Is the circuit stable with and without a resistive load? (i.e. is the negative feedback path gain greater than the positive feedback path gain?).
c) Show that the output admittance of the Howland circuit becomes positive if the op-amp has a finite real gain and $R_1 R_4 = R_2 R_3$. Is the finite gain op-amp Howland circuit stable?

This circuit is called the Howland current source.

HINT: Transform V_s and R_1 into a Norton circuit and use the Negative impedance converter composed by the op-amp and R_2,R_3, R_4 to cancel its output admittance.



Q6 ANSWER:

The ideal NIC consisting of the op-amp, R_2, R_3 and R_4 presents an input admittance $Y_{NIC} = \frac{-R_4}{R_2 R_3}$. If we add this to the $\frac{1}{R_1}$ Norton admittance of the voltage source and R_1 we get:

$$Y_{out} = Y_{NIC} + \frac{1}{R_1} = \frac{-R_4}{R_2 R_3} + \frac{1}{R_1} = \frac{R_2 R_3 - R_1 R_4}{R_1 R_2 R_3}$$

The output admittance of the Howland circuit is zero, and the circuit behaves like an ideal current source if:

$$R_1 R_4 = R_2 R_3$$

This means that the positive and negative feedback gains are equal:

$$\frac{R_3}{R_3 + R_4} = \frac{R_1}{R_1 + R_2}.$$

a) When the Howland source is loaded the magnitude of R_1 decreases, so the negative feedback path gain becomes greater than the positive feedback path gain; the circuit is stable.
b) The Howland source is unstable only when it is not loaded.
c) If the op-amp has a finite gain, the non-inverting amplifier $\{G, R_3, R_4\}$ has a gain of

$$G_{CL} = \frac{G}{1+GH} = \frac{1}{H}\frac{1}{1+1/GH} < \frac{1}{H}, H = \frac{R_3}{R_3 + R_4}$$

By application of the Miller theorem the admittance of the amplifier $\{G, R_3, R_4, R_2\}$ is

$$|Y| = \left| G_2 \left(1 - \frac{G}{1+GH}\right) \right| = G_2 \left(\frac{1}{H}\frac{1}{1+1/GH} - 1 \right) < G_2 \left(\frac{1}{H} - 1 \right)$$

As a result, the output admittance of the Howland source is positive.

$$Y_H = G_1 - |Y| > G_1 - G_2 \left(\frac{1}{H} - 1 \right) = 0 \Rightarrow Y_H > 0$$

Therefore, the Howland source made with a finite gain op-amp is stable.