E2.2 Analogue electronics
Problem sheet 5 (Week 8)

An NPN bipolar transistor is given with $C_{BE} = 5pF, C_{BC} = 1pF, C_{CE} = 0.1pF, V_A = 100V, \beta = 200$.
For the purposes of this problem sheet the transistor is biased at $I_C = 1mA$

QUESTION 1: Use the second form of the Miller Theorem to derive an expression for the input impedance of a common collector amplifier which drives an inductive load L_E. Derive an expression for the frequency at which the input impedance of this amplifier becomes real, and a value for the input impedance at that frequency. Neglect the effect of C_{BC}.

The calculation will be greatly simplified if you use the following substitutions:

$$R = R_\pi, C = C_{BE}, L = (\beta+1)L_E, \tau = RC, \tau_1 = \frac{L}{R}$$

ANSWER

The input impedance of the CE amplifier will be :

$$Z_{in} = (\beta+1)j\omega L_E + \frac{R_\pi}{1+R_\pi j\omega C_{BE}} = \frac{R-\omega^2 RLC + j\omega L}{1+Rj\omega C} = R\frac{1-\omega^2\tau\tau_1 + j\omega\tau_1}{1+j\omega\tau}$$

$$Z_{in} = R\frac{1-\omega^2\tau\tau_1 + j\omega\tau_1}{1+j\omega\tau} = R\frac{1-\omega^2\tau\tau_1 + j\omega\tau_1 - j\omega\tau\left(1-\omega^2\tau\tau_1 + j\omega\tau_1\right)}{1+\omega^2\tau^2} = R\frac{1+j\omega\left(\tau_1-\tau+\omega^2\tau^2\tau_1\right)}{1+\omega^2\tau^2}$$

This becomes real when $\operatorname{Re} Z_{in} = 0 \Rightarrow \tau_1 - \tau + \omega^2\tau^2\tau_1 = 0 \Rightarrow \omega^2 = \frac{\tau-\tau_1}{\tau^2\tau_1} = \frac{1}{\tau\tau_1}\left(1-\frac{\tau_1}{\tau}\right)$

The value of the real resistance is:

$$Z_{in} = \frac{R}{1+\omega^2\tau^2} = \frac{R}{1+\frac{\tau}{\tau_1}\left(1-\frac{\tau_1}{\tau}\right)}, \frac{\tau}{\tau_1} = \frac{R_\pi^2 C_{BE}}{(\beta+1)L_E} \simeq \frac{R_\pi}{L_E\omega_T}, \text{ with } \omega_T = \frac{g_m}{C_{BE}}$$

QUESTION 2: Calculate the value of the load resistance R_L which results in the maximum voltage gain of a Common Emitter amplifier employing Miller Cancellation as a function of the frequency at which Miller cancellation is implemented. You may assume that the source impedance satisfies $R_s << R_\pi$. Sohow that in the limit of very high Miller cancellation frequency the maximum obtainable gain varies as ω^{-2}.

ANSWER

If $R_s << R_\pi$, we can neglect R_π .Miller cancelation implies that at the frequency of interest $\omega^2 C_{BC} L_M = 1$ and C_{BC} does not play a role at all.

The voltage gain is:

$$\left|\frac{v_L}{v_S}\right| = \left|\frac{1}{1+sR_sC_{BE}}\right|\left|\frac{-g_m\left(R_A//R_L\right)}{1+s\left(R_A//R_L\right)C_{CE}}\right| = \left|\frac{1}{1+sR_sC_{BE}}\right|\left|\frac{-g_m\left(\frac{R_AR_L}{R_A+R_L}\right)}{1+s\left(\frac{R_AR_L}{R_A+R_L}\right)C_{CE}}\right| =$$

$$= \frac{1}{\sqrt{1+\omega^2R_s^2C_{BE}^2}}\frac{g_m\left(\frac{R_AR_L}{R_A+R_L}\right)}{\sqrt{1+\omega^2\left(\frac{R_AR_L}{R_A+R_L}\right)^2C_{CE}^2}}$$

We need to maximize the quantity $\frac{x}{\sqrt{1+x^2}}$ with $x = \frac{R_AR_L}{R_A+R_L}C_{CE}$, since the only thing we can vary is R_L

$$\frac{d}{dR_L}\frac{x}{\sqrt{1+x^2}} = \frac{dx}{dR_L}\frac{d}{dx}\frac{x}{\sqrt{1+x^2}} = \left(\frac{\sqrt{1+x^2}-\frac{x^2}{\sqrt{1+x^2}}}{1+x^2}\right)\frac{d}{dR_L}\frac{R_AR_L}{R_A+R_L} =$$

$$= \frac{1}{\left(1+x^2\right)^2}\frac{\left(R_A+R_L\right)R_A - R_AR_L}{\left(R_A+R_L\right)^2} = \frac{R_A^2}{\left(1+x^2\right)^2\left(R_A+R_L\right)^2} > 0$$

This is a monotonically increasing with R_L function and the maximum gain is therefore:

$$\max\left|\frac{v_L}{v_S}\right| = \left|\frac{1}{1+sR_sC_{BE}}\right|\left|\frac{g_mR_A}{1+sR_AC_{CE}}\right| = \frac{1}{\sqrt{1+\omega^2R_s^2C_{BE}^2}}\frac{g_mR_A}{\sqrt{1+\omega^2R_A^2C_{CE}^2}} \to \frac{g_m}{\omega^2R_sC_{BE}C_{CE}}$$

QUESTION 3: A negative feedback amplifier is built with a dominant pole forward amplifier and feedback gain constant with frequency. Prove that the closed loop amplifier is also a dominant pole amplifier whose gain-bandwidth product is constant and equal to the gain-bandwidth product of the forward amplifier.

ANSWER

The forward gain is $G(s)=\frac{G_0}{1+s\tau}$. The reverse gain is $H(s)$. The closed loop gain is:

$$A(s)=\frac{G(s)}{1+G(s)H}=\frac{\frac{G_0}{(1+s\tau)}}{1+\frac{G_0}{(1+s\tau)}H}=\frac{G_0}{1+G_0H+s\tau}=\frac{G_0}{1+G_0H}\frac{1}{1+s\tau/(1+G_0H)}=$$

$$=\frac{G'}{1+s\tau'},G'=\frac{G_0}{1+G_0H},\tau'=\frac{\tau}{1+G_0H}\Rightarrow G'/\tau'=G_0/\tau \text{ QED}$$

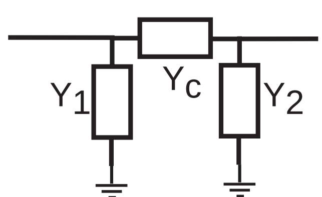
QUESTION 4: (Tutorial Question Week 8-9)

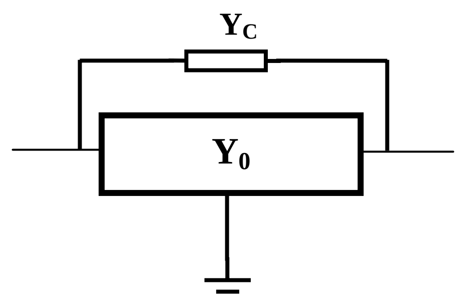
Show that the admittance matrix of the "Pi" network shown on the left below is:

$$Y=\begin{bmatrix} Y_1+Y_C & -Y_C \\ -Y_C & Y_2+Y_C \end{bmatrix}$$

Connect an admittance Y_C between the input and output of a network with the admittance matrix: $\mathbf{Y}_0=\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$. Use KCL to show that the resulting circuit (shown in the figure below) has an admittance matrix equal to: $\mathbf{Y}'=\begin{bmatrix} y_{11}+Y_C & y_{12}-Y_C \\ y_{21}-Y_C & y_{22}+Y_C \end{bmatrix}$

The admittance matrix of a 2-port is defined as: $Y_{ij}=\left.\frac{\partial i_i}{\partial v_j}\right|_{\delta v_{k\neq j}=0}$ i.e. the current in port "i" is measured by an ideal ammeter as a voltage differential is applied on port "j".





ANSWER

By definition,

$$y_{11} = \left.\frac{\partial i_1}{\partial v_1}\right|_{v_2=0} = Y_1 + Y_C \qquad y_{12} = \left.\frac{\partial i_1}{\partial v_2}\right|_{v_1=0} = -Y_C$$

$$y_{21} = \left.\frac{\partial i_2}{\partial v_1}\right|_{v_2=0} = -Y_C \qquad y_{22} = \left.\frac{\partial i_2}{\partial v_2}\right|_{v_1=0} = Y_2 + Y_C$$

A network with only Yc between ports 1 and 2 has an admittance matrix $\mathbf{Y}' = \begin{bmatrix} Y_C & -Y_C \\ -Y_C & Y_C \end{bmatrix}$. The network on the right consists of two networks connected in parallel, so its admittance matrix is the sum of the individual admittance matrices.(You can see this by taking the derivatives)

It follows that the total admittance matrix is $\mathbf{Y}' = \begin{bmatrix} y_{11} + Y_C & y_{12} - Y_C \\ y_{21} - Y_C & y_{22} + Y_C \end{bmatrix}$