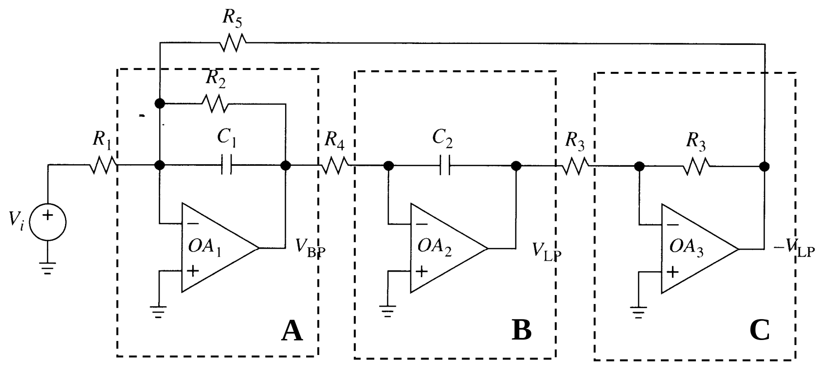
E2.2 Analogue electronics
Problem sheet 7 (Week 9)

Q1: (Tutorial question week 10/11)

Draw a flow graph for the biquadratic filter below, assuming the op-amps are ideal. Write an expression for the three transfer functions $G_{LP} = \frac{V_{LP}}{V_i}$, $G_{BP} = \frac{V_{BP}}{V_i}$ in terms of the block transimpedance gains and the connecting conductances.

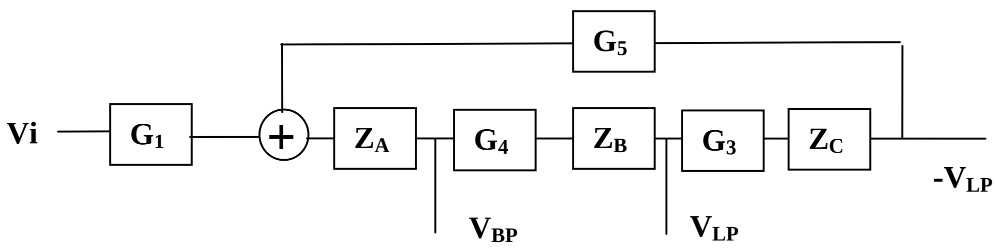


ANSWER

Since the op-amps are ideal the networks in the shaded boxes are ideal transimpedance amplifiers of gains

$Z_A = \frac{-R_2}{1+sR_2C_1}$, $Z_B = \frac{-1}{sC_2}$, $Z_C = -R_3$. The input terminals are virtual grounds, and sum currents .

Therefore, the flow graph can be drawn as:



With $G_n = 1/R_n$.

It follows that

$$G_{BP} = \frac{V_{BP}}{V_i} = \frac{G_1 Z_A}{1 - Z_A G_4 Z_B G_3 Z_C G_5}$$

$$G_{LP} = \frac{V_{BP}}{V_i} = \frac{G_1 Z_A G_4 Z_B}{1 - Z_A G_4 Z_B G_3 Z_C G_5} = G_{BP} G_4 Z_B$$

Q2. You are given the Common Emitter Y parameters of a Bipolar transistor. Calculate the Common Base H parameters of this transistor.

ANSWER:

Can be done directly, or in 2 steps, $Y_{CE} \to Y_{CB} \to H_{CB}$, or directly. We the second calculation, since the first is in the notes:

The voltages and currents in the two representations are related as:

$$i_{CB1} = i_E = -(i_B + i_C) = -i_{CE1} - i_{CE2}$$
$$i_{CB2} = i_C = i_{CE2}$$
$$v_{CB1} = v_{EB} = -v_{BE} = -v_{CE1}$$
$$v_{CB2} = v_{CB} = v_{CE} - v_{BE} = -v_{CE1} + v_{CE2}$$

We write the definition of the CB h matrix:

$$\begin{bmatrix} v_{CB1} \\ i_{CB2} \end{bmatrix} = h_{CB} \begin{bmatrix} i_{CB1} \\ v_{CB2} \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_{CE1} \\ v_{CE2} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} i_{CE1} \\ i_{CE2} \end{bmatrix} = h_{CB}\left(\begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} v_{CE1} \\ v_{CE2} \end{bmatrix} + \begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} i_{CE1} \\ i_{CE2} \end{bmatrix}\right) \Rightarrow$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} v_{CE1} \\ v_{CE2} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} y_{CE} \begin{bmatrix} v_{CE1} \\ v_{CE2} \end{bmatrix} = h_{CB}\left(\begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} v_{CE1} \\ v_{CE2} \end{bmatrix} + \begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix} y_{CE} \begin{bmatrix} v_{CE1} \\ v_{CE2} \end{bmatrix}\right) \Rightarrow$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} y_{CE} = h_{CB}\left(\begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix} y_{CE}\right) \Rightarrow$$

$$\begin{bmatrix} -1 & 0 \\ y_{21} & y_{22} \end{bmatrix} = h_{CB} \begin{bmatrix} -y_{11} - y_{21} & -y_{12} - y_{22} \\ 1 & -1 \end{bmatrix} \Rightarrow$$

$$h_{CB} = \begin{bmatrix} -1 & 0 \\ y_{21} & y_{22} \end{bmatrix}\begin{bmatrix} -y_{11} - y_{21} & -y_{12} - y_{22} \\ 1 & -1 \end{bmatrix}^{-1} = \frac{1}{(y_{11} + y_{12} + y_{21} + y22)}\begin{bmatrix} -1 & 0 \\ y_{21} & y_{22} \end{bmatrix}\begin{bmatrix} -1 & y_{12} + y_{22} \\ -1 & -y_{11} - y_{21} \end{bmatrix}$$

3. You are given the common source Y matrix of a FET. Compute its transmission matrix in common source configuration.

ANSWER:

The transmission matrix is:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1/g_{21} & -1/y_{21} \\ 1/z_{21} & -1/h_{21} \end{bmatrix}$$

The y parameters are given. The rest are:

$$g_{21} = \frac{-y_{21}}{y_{22}}, z_{21} = \frac{-y_{21}}{\det y}, h_{21} = \frac{y_{21}}{y_{11}}$$

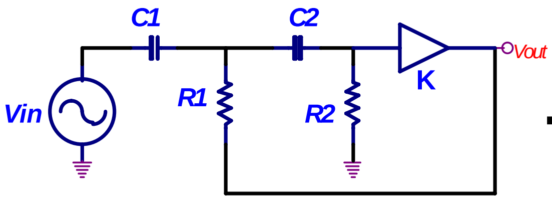
4. You are given the common source transmission matrix of a FET. Compute the common Gate and common Drain transmission matrices.

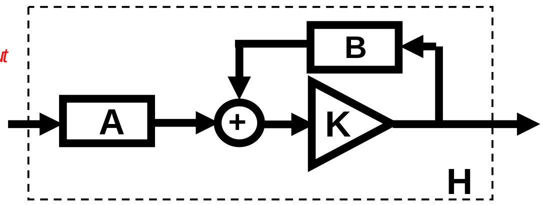
ANSWER

Same logic as Q2.

5. Solve the high pass sallen-key filter.

ANSWER:





Derive A and B by superposition:

Convert V_{in}, C_1, R_1 into a Thevenin equivalent: $V_T = \dfrac{sR_1C_1}{1+sR_1C_1}, Z_T = \dfrac{R_1}{1+sR_1C_1}$

And calculate the resultant voltage divider:

$$A = \frac{s^2R_1C_1R_2C_2}{s^2R_1C_1R_2C_2 + s\left(R_1C_1 + R_2C_2 + R_1C_2\right) + 1} = \frac{s^2\tau_1\tau_2}{D}$$

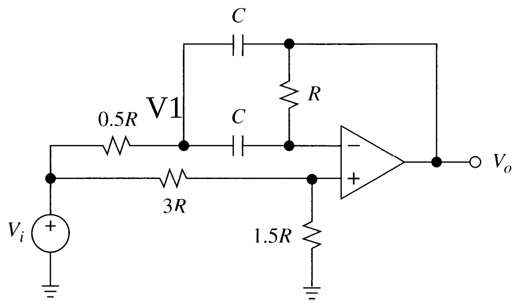
$$B = \frac{sR_2C_2}{s^2R_1C_1R_2C_2 + s\left(R_1C_1 + R_2C_2 + R_1C_2\right) + 1} = \frac{s\tau_2}{D}$$

$$D = s^2\tau_1\tau_2 + s\left(\tau_1 + \tau_2 + \tau_{12}\right) + 1, \tau_1 = R_1C_1, \tau_2 = R_2C_2, \tau_{12} = R_1C_2$$

$$H(s) = \frac{AK}{1-BK} = \frac{s^2\tau_1\tau_2K}{D - Ks\tau_2} = \frac{s^2\tau_1\tau_2K}{s^2\tau_1\tau_2 + s\left(\tau_1 + \tau_2(1-K) + \tau_{12}\right) + 1}$$

6. Solve the Deliyannis-Friend All pass filter

ANSWER:



Evaluate the (3R, 1.5R) voltage divider to find that $V_- = V_+ = V_i / 3$

Write nodal equations:

At V_1:

$$2G\left(V_i - V_1\right) + \left(V_o - V_1\right)sC + \left(V_- - V_1\right)sC = 0 \Rightarrow 2G\left(V_i - V_1\right) + \left(V_o - V_1\right)sC + \left(V_i / 3 - V_1\right)sC = 0 \Rightarrow$$
$$\left(2G + 2sC\right)V_1 - sCV_o = \left(2G + sC / 3\right)V_i\,, \quad G = 1/R$$

At V_-:

$$sC\left(V_1 - V_-\right) + \left(V_o - V_-\right)G = 0 \Rightarrow sC\left(V_1 - V_i / 3\right) + \left(V_o - V_i / 3\right)G = 0 \Rightarrow$$
$$sCV_1 + GV_o - \left(sC + G\right)V_i / 3 = 0 \Rightarrow sCV_1 + GV_o = V_i\left(sC + G\right)/3$$

Put all together:

$$\begin{bmatrix} \left(2G + 2sC\right) & -sC \\ sC & G \end{bmatrix} \begin{bmatrix} V_1 \\ V_o \end{bmatrix} = \begin{bmatrix} 2G + sC/3 \\ \left(sC + G\right)/3 \end{bmatrix} V_i \Rightarrow$$
$$\frac{V_o}{V_i} = \frac{G\left(2G + 2sC\right)\left(sC + G\right)/3 - sC\left(2G + sC/3\right)}{G\left(2G + 2sC\right) + s^2C^2}$$

Multiply numerator and denominator by $3R^2$ and call $\tau = RC$ to get:

$$\frac{V_o}{V_i} = \frac{1}{3}\frac{\left(2 + 2s\tau\right)\left(s\tau + 1\right) - s\tau\left(6 + s\tau\right)}{\left(2 + 2s\tau\right) + s^2\tau^2} = \frac{1}{3}\frac{s^2\tau^2 - 2s\tau + 2}{s^2\tau^2 + 2s\tau + 2}$$