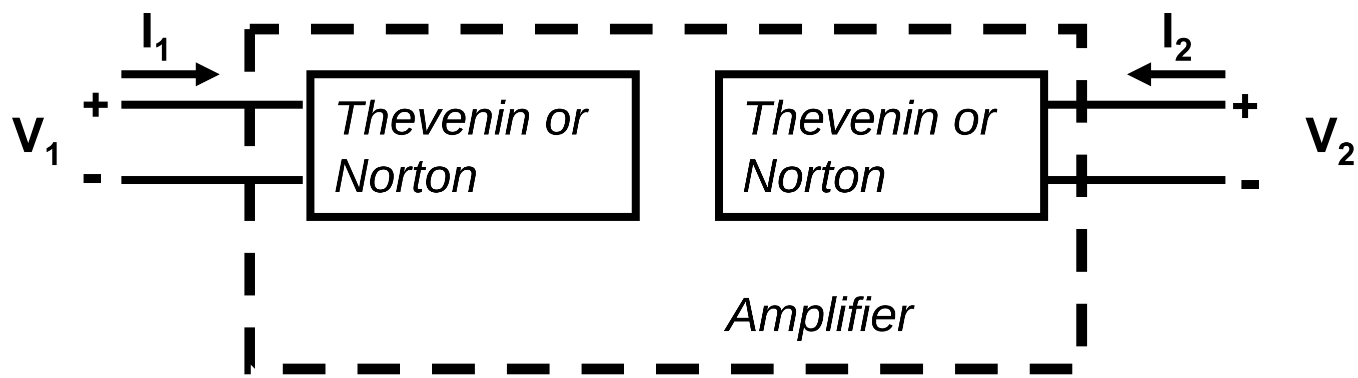
Revision of intermediate electronics

2ports, feedback and filters

Generalised Thevenin + Norton Theorems:
2-port parameters

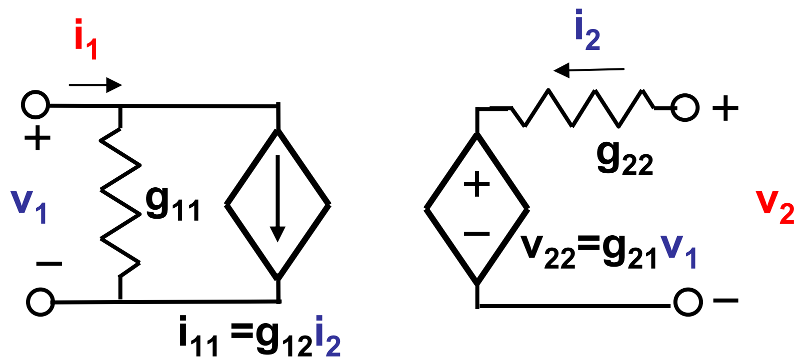
- Amplifiers, filters etc have input and output
- Input can be voltage or current
- Output can be voltage or current
- By convention current positive into positive terminal
- Negative terminals usually considered connected together
- General form of amplifier or filter:



The voltage amplifier – G parameters

Also called the reverse hybrid parameters

Norton



Thevenin

Formal description:

g_{11}: Input admittance

g_{12}: Reverse current gain

g_{21}: Voltage gain

g_{22}: Output impedance

$$\left.\begin{aligned} i_1 &= g_{11}v_1 + i_{11} = g_{11}v_1 + g_{12}i_2 \\ v_2 &= v_{22} + g_{22}i_2 = g_{21}v_1 + g_{22}i_2 \end{aligned}\right\} \Rightarrow$$

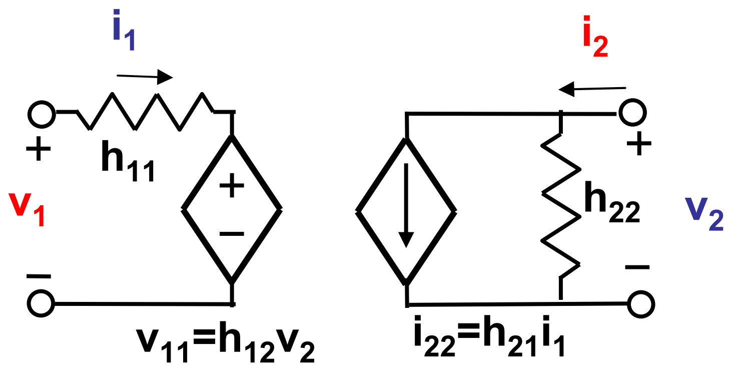
$$\begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \mathbf{G} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix}$$

- *A voltage amplifier exhibits non-zero reverse current gain!*
- *The port-reversed amplifier is a current “amplifier”*
- *Reverse path has less than unity **power** gain.*
- *If the reverse current gain is zero, the amplifier is called* **Unilateral**

The current amplifier – H parameters

Also called the hybrid parameters

Thevenin



Norton

Formal description

h_{11}: Input impedance

h_{12}: Reverse voltage gain

h_{21}: Current gain

h_{22}: Output admittance

$$\left.\begin{aligned} v_1 &= h_{11} i_1 + v_{11} = h_{11} i_1 + h_{12} v_2 \\ i_2 &= i_{22} + h_{22} v_2 = h_{21} i_1 + h_{22} v_2 \end{aligned}\right\} \Rightarrow$$

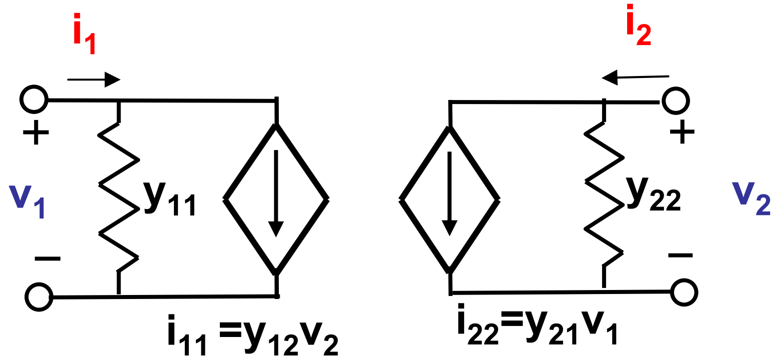
$$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \mathbf{H} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$$

- *A current amplifier exhibits a reverse voltage gain!*
- *The port-reversed amplifier is a voltage "amplifier"*
- *Reverse path has less than unity **power** gain.*
- *If the reverse voltage gain is zero, the amplifier is called* **Unilateral**

The transconductance amplifier – Y parameters

Also called the short circuit parameters

Norton



Norton

Formal description

y_{11}: Input admittance

y_{12}: Reverse admittance gain

y_{21}: trans-admittance (gain)

y_{22}: Output admittance

$$\left.\begin{aligned} i_1 &= y_{11}v_1 + i_{11} = y_{11}v_1 + y_{12}v_2 \\ i_2 &= i_{22} + y_{22}v_2 = y_{21}v_1 + y_{22}v_2 \end{aligned}\right\} \Rightarrow$$

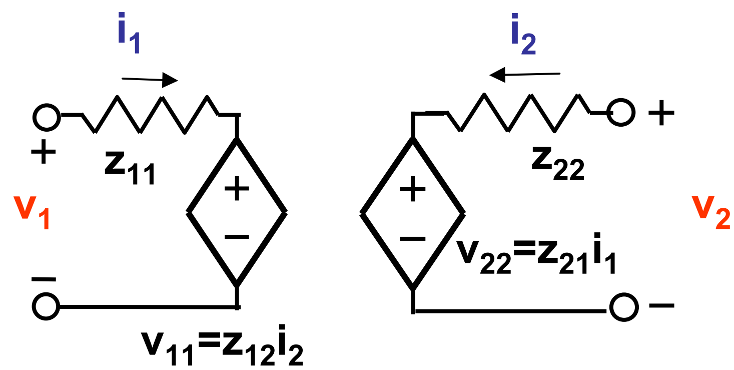
$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \mathbf{Y} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

- *A transconductance amplifier exhibits a reverse transconductance gain!*
- *The port-reversed amplifier is also a transconductance “amplifier”*
- *Reverse path has less than unity* ***power*** *gain.*
- *If the reverse gain is zero, the amplifier is called* **Unilateral**

The transresistance amplifier – Z parameters

Also called the open circuit parameters

Thevenin



Thevenin

Formal description

z_{11}: Input impedance

z_{12}: Reverse impedance gain

z_{21}: transimpedance gain

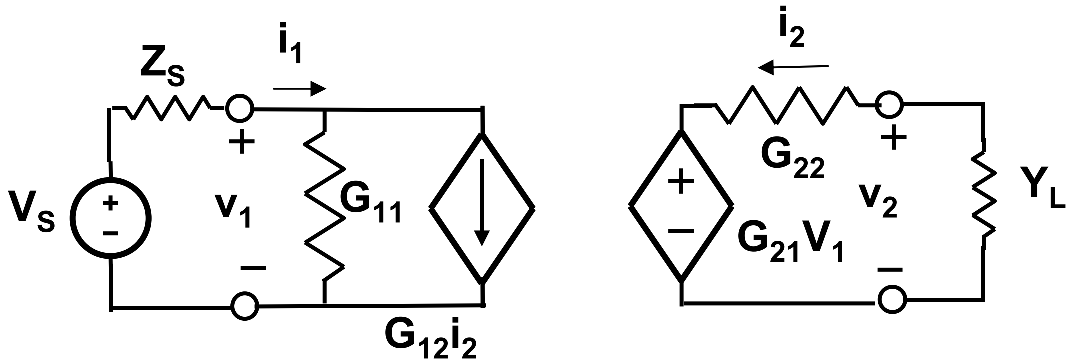
z_{22}: Output impedance

$$\left.\begin{aligned} v_1 &= z_{11} i_1 + v_{11} = z_{11} i_1 + z_{12} i_2 \\ v_2 &= v_{22} + z_{22} i_2 = z_{21} i_1 + z_{22} i_2 \end{aligned}\right\} \Rightarrow$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \mathbf{Z} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

- *A transresistance (also called a transimpedance) amplifier exhibits a reverse transimpedance gain!*
- *The port-reversed amplifier is also a transresistance "amplifier"*
- *Reverse path has less than unity **power** gain.*
- *If the reverse gain is zero, the amplifier is called* **Unilateral**

Gain of a fully loaded voltage amplifier



We start with the amplifier definition,
plus the source-load boundary conditions:

$$\left.\begin{aligned} i_1 &= g_{11}v_1 + g_{12}i_2 \\ v_2 &= g_{21}v_1 + g_{22}i_2 \\ v_1 &= v_s - i_1 Z_s \\ i_2 &= -v_2 Y_L \end{aligned}\right\} \Rightarrow \begin{cases} i_1 = g_{11}\left(v_s - i_1 Z_s\right) - g_{12}v_2 Y_L \\ v_2 = g_{21}\left(v_s - i_1 Z_s\right) - g_{22}v_2 Y_L \end{cases}$$

After a lot of algebra we conclude that:

$$\frac{v_2}{v_s} = \frac{g_{21}}{1 + g_{11}Z_S + g_{22}Y_L + \Delta_g Y_L Z_s}\ , \Delta_g = g_{11}g_{22} - g_{21}g_{12}$$

Cascade connection: Transmission Parameters

In a cascade connection,

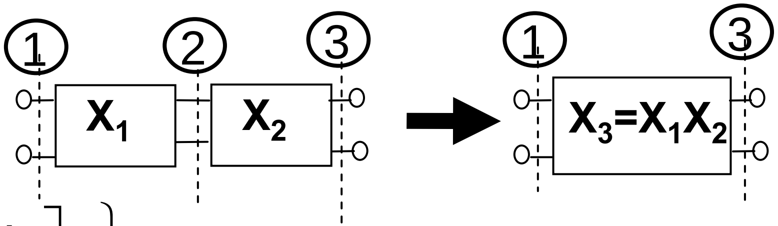
- V_1 of network X_2= V_2 of network X_1
- I_1 of network X_2 = -I_2 of network X_1

We can define a new set of parameters so that we have a simple way to calculate the response of cascades of amplifiers.

A suitable definition is:

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

With this definition, the ABCD parameters of a cascade of two networks are found from the matrix product of the individual ABCD matrices ports labelled for clarity):



$$\left.\begin{array}{l} \begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix} \\ \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} v_3 \\ -i_3 \end{bmatrix} \end{array}\right\} \Rightarrow \begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} v_3 \\ -i_3 \end{bmatrix} \Rightarrow \begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} \begin{bmatrix} v_3 \\ -i_3 \end{bmatrix}$$

Transmission (or ABCD) parameters (2)

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix}$$

- ***Note the sign of i_2 and also the reverse sense of signal flow. The sign is chosen so the ABCD matrix of a cascade of two networks is just the matrix product of the individual ABCD matrices (compare this to the messy loading calculation before!)***
- ***The reverse sense of signal flow is to keep the matrix finite if an amplifier is unilateral.***
- ***The conversion from, say, Y to ABCD follows the same logic as the Y(H) calculation:***

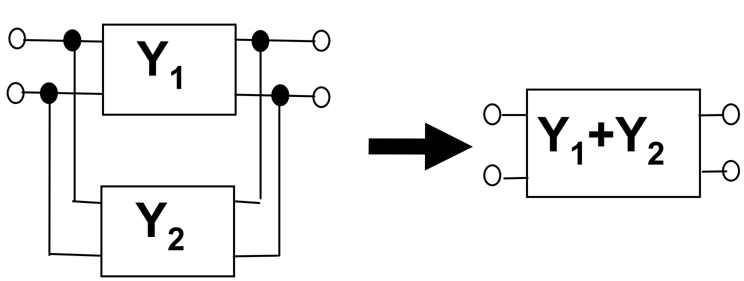
$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_2 \\ -i_2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ Y_{11} & Y_{12} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -Y_{21} & -Y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{-1}{Y_{21}} \begin{bmatrix} Y_{22} & 1 \\ \Delta_Y & Y_{11} \end{bmatrix}, \ \Delta_y = Y_{11}Y_{22} - Y_{21}Y_{12}$$

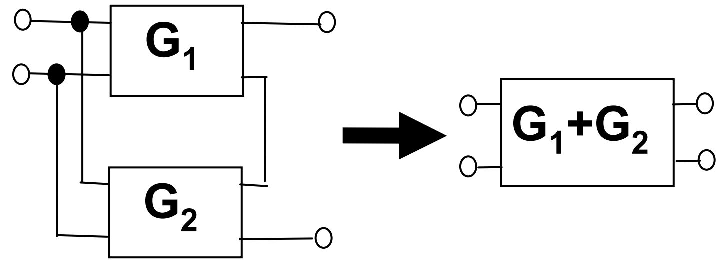
Note that all ABCD parameters are inversely proportional to the gain. This is the reason for formally choosing port 2 as the input port.
The intuitive choice of input at port 1 would make all parameters inversely proportional to the reverse gain, which is small, and often not very accurately determined.

2-port Connection rules summary

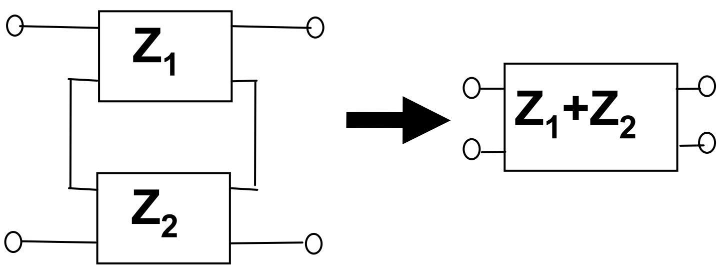
For the exact calculation of circuit interconnections we can use 2-port matrix algebra:



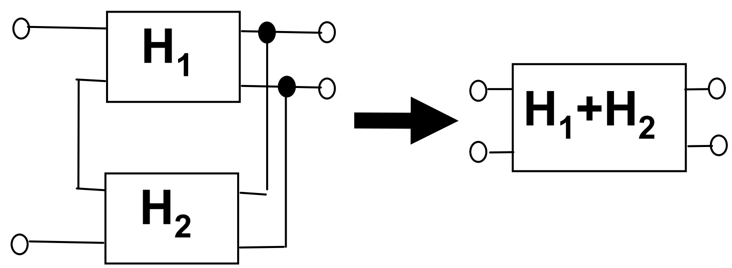
shunt-shunt: add Y matrices



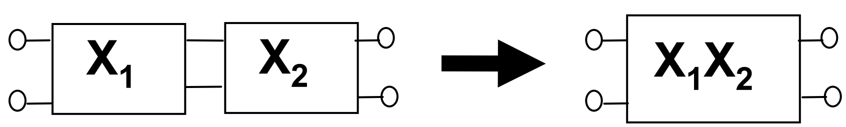
shunt-series: add G matrices



series-series: add Z matrices



series-shunt: add H matrices



cascade connection: multiply ABCD matrices

more on 2-port parameters

We can determine 2-port parameters from the definitions.
For example, the Y parameter description states that:

$$i_1 = y_{11}v_1 + y_{12}v_2$$
$$i_2 = y_{21}v_1 + v_{22}v_2$$

These relations imply that the y parameters are partial derivatives:

$$y_{11} = \left.\frac{\partial i_1}{\partial v_1}\right|_{v_2=0} \qquad y_{12} = \left.\frac{\partial i_1}{\partial v_2}\right|_{v_1=0}$$
$$y_{21} = \left.\frac{\partial i_2}{\partial v_1}\right|_{v_2=0} \qquad y_{22} = \left.\frac{\partial i_2}{\partial v_2}\right|_{v_1=0}$$

Note that these are small signal parameters, so, e.g. v_1*=0 means that* v_1 *is kept constant.*

The y_{21} *parameter of a transistor is the familiar* **transconductance.**

Input and output impedance of an amplifier

- We often use "Impedance" or "Admittance" to imply which derivative we have in mind. The **input impedance** is the **z11 or g11** while the **input admittance** the **y11** or **h11** parameter.

- For example, if the output is open circuited:

$$Z_{input} = Z_{in}\Big|_{Y_L=0} = \frac{\partial v_1}{\partial i_1}\Bigg|_{i_2=0} = z_{11} = 1/g_{11}$$

- On the other hand, if the output is shorted, then:

$$Y_{input} = Y_{in}\Big|_{Z_L=0} = \frac{\partial i_1}{\partial v_1}\Bigg|_{v_2=0} = y_{11} = 1/h_{11}$$

- From our discussion about parameter conversions we know that:

$$Z_{11} = \left(\mathbf{Y}^{-1}\right)_{11} = \frac{y_{22}}{y_{11}y_{22} - y_{21}y_{12}} \neq \frac{1}{y_{11}}$$

- **Note that only if the amplifier is unilateral (y_{12}=0) we have z_{11}=1/y_{11}**
- Similarly the output impedance of an amplifier depends on whether the amplifier is driven by a voltage or a current source, and indeed, on the value of the source impedance in the general case.
- The argument can be reversed: **If the input or output impedance of an amplifier does not depend on the load or the source impedance respectively, then the amplifier is necessarily unilateral**

Amplifiers: modelling summary

Name / Representation	Parameters	Input	Output	Forward gain	Reverse gain
Voltage	G	Norton	Thevenin	Voltage	Current
Current	H	Thevenin	Norton	Current	Voltage
Transconductance	Y	Norton	Norton	Admittance	Admittance
Transimpedance	Z	Thevenin	Thevenin	Impedance	Impedance

Name / Representation	Input	Output	Ideal form	Terminal impedance			
				Ideal		Real	
				Input	Output	Input	Output
Voltage	V	V	VCVS	∞	0	High	Low
Current	I	I	CCCS	0	∞	Low	High
Transconductance	V	I	VCCS	∞	∞	High	High
Transimpedance	I	V	CCVS	0	0	Low	Low

***Notes**:*

1. Choice of representation is **arbitrary**
2. Representation emphasises the intended function
3. *Can convert one representation into any other by Thevenin←→Norton transforms*

Conversion between amplifier representations

- Some are obvious matrix inversions from the matrix equations:

$$\mathbf{Z} = \mathbf{Y}^{-1}$$

$$\mathbf{G} = \mathbf{H}^{-1}$$

Recall that the inverse of a 2x2 matrix A is:

$$\mathbf{A}^{-1} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{-1} = \frac{1}{\Delta_a} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

- Other conversions, e.g. to express y in terms of h,

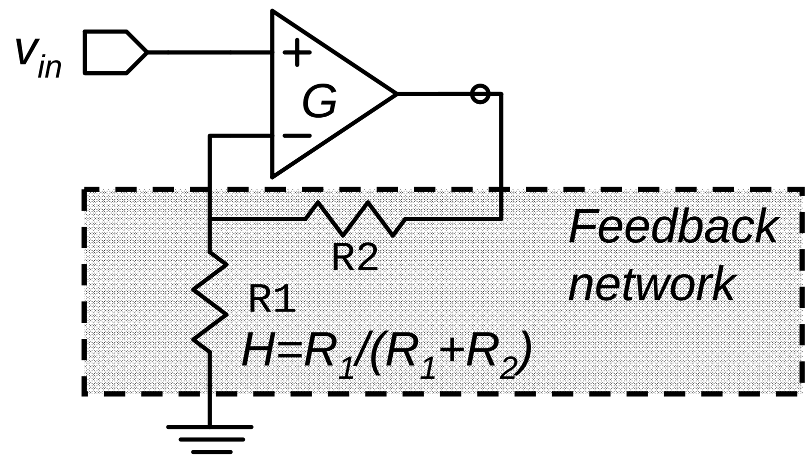
start with the definitions and express the variables of the y description in terms of h parameters. The resulting equation is an identity valid for all values of the h description independent variables.

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix} = \mathbf{Y} \begin{bmatrix} h_{11} & h_{12} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix} \Rightarrow$$

$$\mathbf{Y} = \begin{bmatrix} 1 & 0 \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} h_{11} & h_{12} \\ 0 & 1 \end{bmatrix}^{-1} = \cdots = \frac{1}{h_{11}} \begin{bmatrix} 1 & -h_{12} \\ h_{21} & \Delta_h \end{bmatrix}, \ \Delta_h = h_{11}h_{22} - h_{12}h_{21}$$

All this can also be done by re-arranging the terms in the equations. This may be easier at the beginning.

The non-inverting amplifier



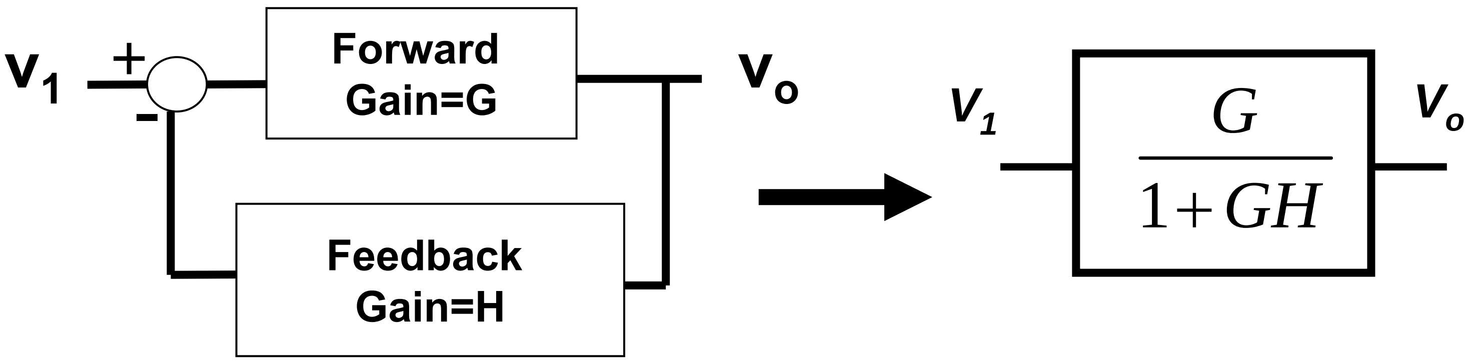
Assume finite op-amp gain G. Treat the network connecting the output and the input as an ideal "amplifier" with gain $H=R_1/(R_1+R_2)$ from output to input

$$v_{out} = G\left(v_{in} - \frac{R_1}{R_1+R_2}v_{out}\right) \Rightarrow v_{out} = \frac{Gv_{in}}{1+G\left(\frac{R_1}{R_1+R_2}\right)} = \frac{G}{1+GH}v_{in} \Rightarrow \lim_{G\to\infty}\frac{v_{out}}{v_{in}} = \frac{1}{H} = 1+\frac{R_2}{R_1}$$

Negative feedback: A "control systems" perspective

- The signals on the network must be self-consistent

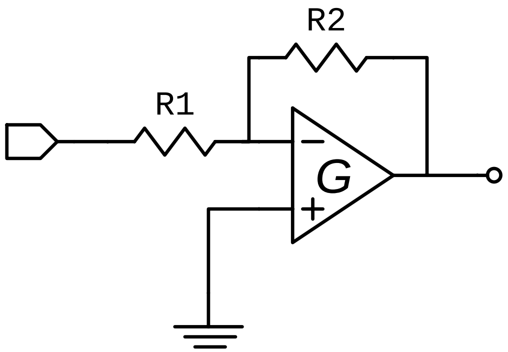
$$v_o = G\left(v_1 - v_o H\right) \Rightarrow v_0 = \frac{v_1 G}{1+GH}$$



If GH is large, Taylor expansion gives: $$\frac{v_o}{v_i} = \frac{1}{H}\left(1 - \frac{1}{GH} + \frac{1}{\left(GH\right)^2} - \cdots\right)$$

GH is called the ***LOOP GAIN*** *G_L*

The inverting amplifier



$$v_o = G\left(-Kv_1 - v_o H\right) \Rightarrow v_0 = -v_1 \frac{KG}{1+GH}$$

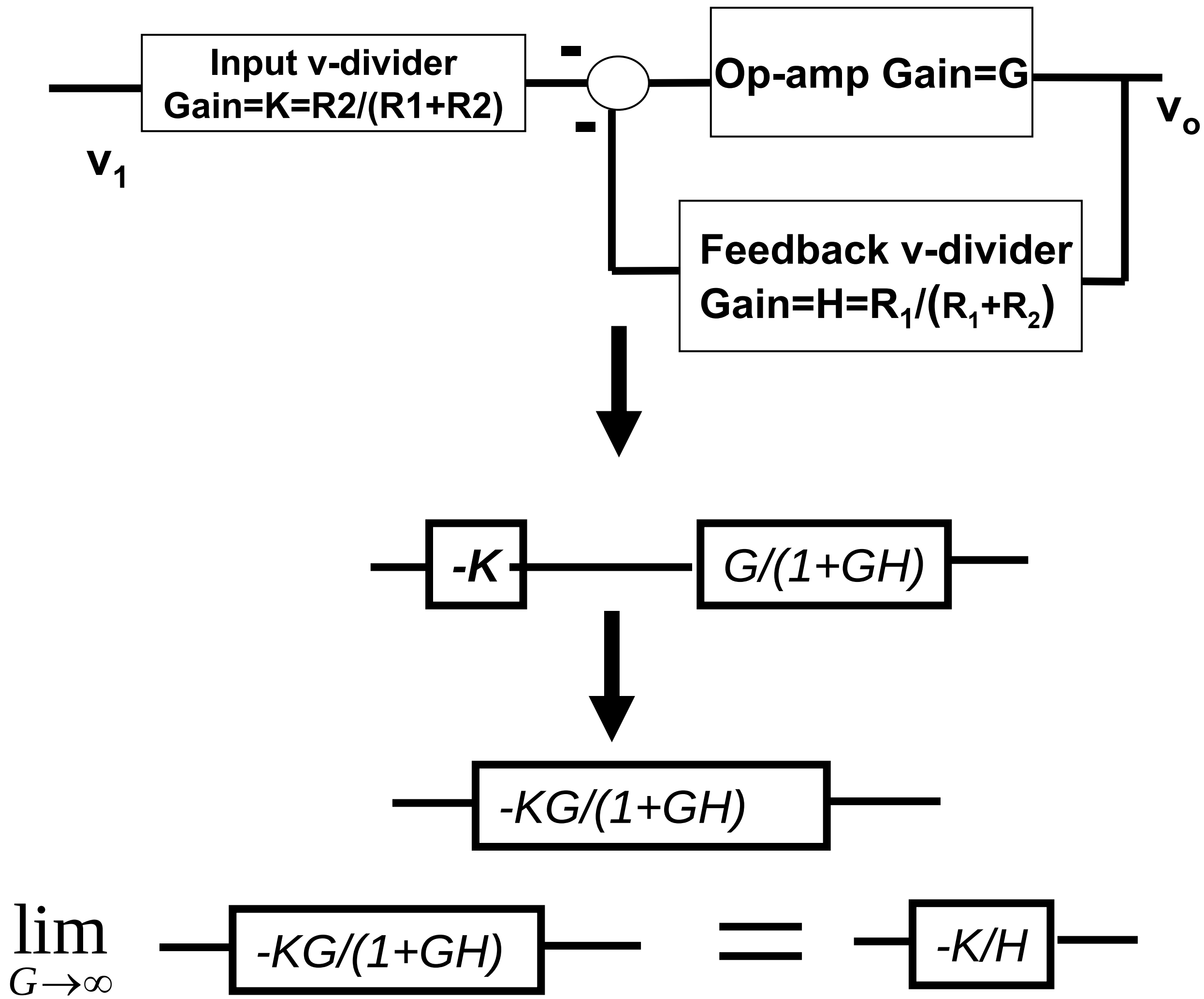
$$\lim_{G\to\infty} v_o = \lim_{G\to\infty} \frac{-v_1 K}{H}\left(1 - \frac{1}{GH} + \ldots\right) = -v_1 \frac{R_2}{R_1}$$

$$K = R_2/\left(R_1 + R_2\right)\ ,\ H = R_1/\left(R_1 + R_2\right)$$

Note: v_{out} and v_{in} are applied by superposition on the circuit. This way we obtain the 2 voltage dividers K,H

There are two negative signs on the summing junction, since both forward and feedback signals are applied on the inverting input

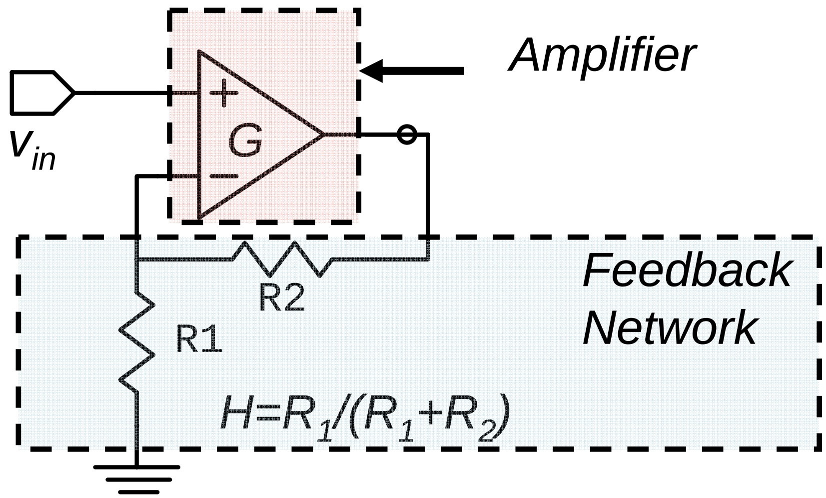
The inverting amplifier:
A simpler way to calculate



Feedback in electronics

- There is **both** a voltage and a current at every terminal
- Precise definitions of measurements:
  - **Voltage** is measured **with voltmeters**. Voltmeters are **connected in parallel** to the circuit, and have infinite internal resistance (VM draw **no current**)..
  - **Current** is measured with **ammeters**. Ammeters are **connected in series** to the circuit and have zero internal resistance (AM develop no voltage).
- There are 4 ways to implement electronic feedback:
  - We may "**sample**" (measure) the output:
    - **Voltage**, by connecting the input (port 2!) of the FB network in shunt (**parallel**)
    - **Current**, by connecting the input (port 2!) of the FB network in **series**
  - We can then "**mix**" (feed back) the signal to the input as:
    - **Voltage**, by connecting the output (port 1!) of the FB network in **series**
    - **Current** by connecting the output (port 1!) of the FB network in shunt (**parallel**)
- **Exact** description of electronic feedback involves **2-port matrix addition**.
- This is very tedious, we usually use approximations.

The non-inverting amplifier: series – shunt feedback

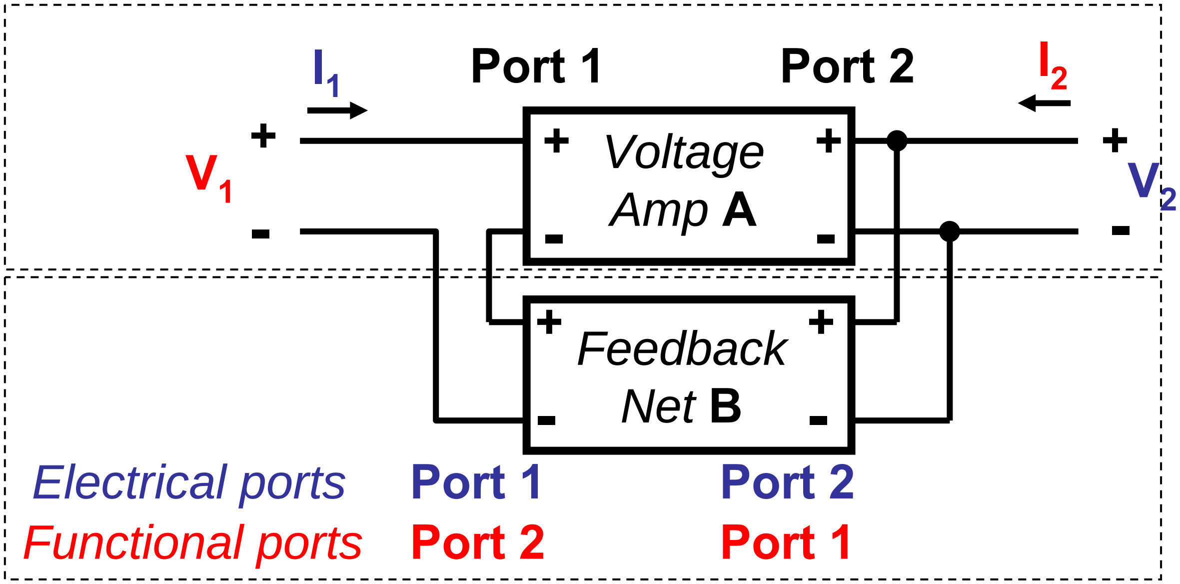


The op-amp acts like a voltage amplifier
The feedback network samples the output voltage, voltage divides it and feeds back a voltage into the input, so that v_{in} is the sum of input and fed back v.

The feedback network shares input current and output voltage with the op-amp

Feedback on voltage amplifiers

Series-Shunt connection: Add H parameters



Function of feedback net:
Measure output V
Correct (mix) input V
i.e. it improves a G-amp

Shared electrical variables:

Input: I_1 , V_2

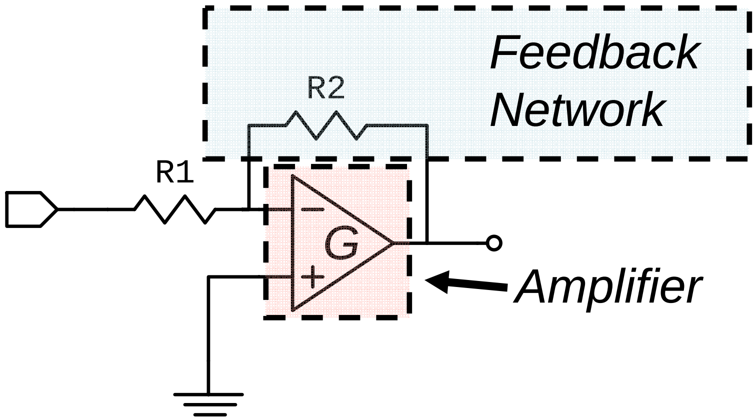
Output: V_1 , I_2

The feedback network is functionally a voltage amplifier from Port2 → Port1
Electrically both networks must be treated as current amplifiers P1→P2
to account for the shared (input) electrical variables.
In the calculation we consider the ELECTRICAL description:

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_{1A} \\ I_{2A} \end{bmatrix} + \begin{bmatrix} V_{1B} \\ I_{2B} \end{bmatrix} = \mathbf{H}_{\mathbf{A}} \begin{bmatrix} I_{1A} \\ V_{2A} \end{bmatrix} + \mathbf{H}_B \begin{bmatrix} I_{1B} \\ V_{2B} \end{bmatrix} = \left(\mathbf{H}_{\mathbf{A}} + \mathbf{H}_{\mathbf{B}}\right) \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \mathbf{H}_{\mathbf{A+B}} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

- *Add H parameter representations of amplifier and feedback network*
- *Convert back to G parameter representation for composite V-amp.*

The inverting amplifier: Shunt – Shunt feedback



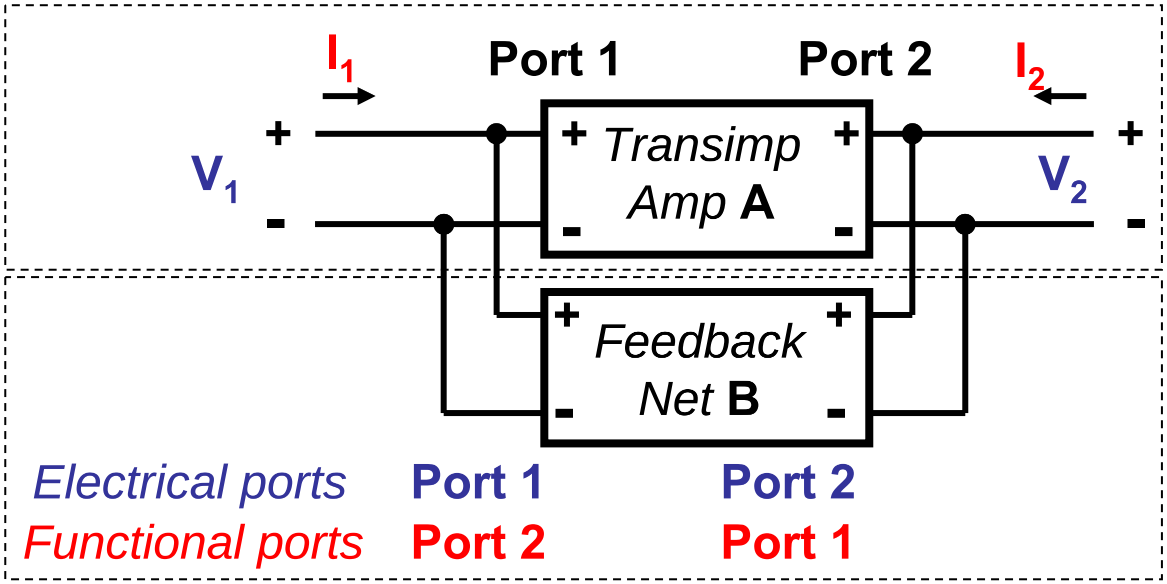
Amplifier and feedback network have identical input and output voltages

The feedback network samples the output **voltage** and contributes a **current** to correct the input. The amplifier G functions as a CCVS (but this should not confuse us, the representation is arbitrary!)

Since the amplifier and the feedback network share voltages they must be treated as transconductors!

Feedback on transimpedance amplifiers

The Shunt-Shunt connection: Add Y parameters



Function of feedback net:
Measure output V
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Shared electrical variables:

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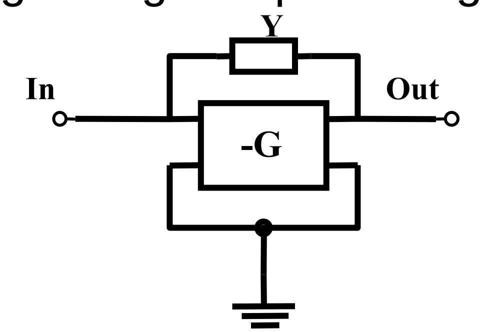
The feedback network is functionally a transconductance amplifier from Port2 → Port1
Electrically both networks must be treated as transconductance amplifiers P1→P2
to account for the shared (input) electrical variables.
In the calculation we consider the ELECTRICAL description:

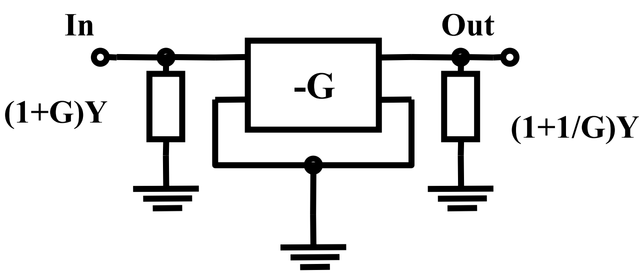
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_{1A} \\ I_{2A} \end{bmatrix} + \begin{bmatrix} I_{1B} \\ I_{2B} \end{bmatrix} = \mathbf{Y_A} \begin{bmatrix} V_{1A} \\ V_{2A} \end{bmatrix} + \mathbf{Y_B} \begin{bmatrix} V_{1B} \\ V_{2B} \end{bmatrix} = \left(\mathbf{Y_A} + \mathbf{Y_B}\right) \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \mathbf{Y_{A+B}} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

- *Add Y parameter representations of amplifier and feedback network*
- *Convert to Z parameter representation for composite Z-amp*

The Miller theorem: shunt-shunt feedback

- Consider a shunt admittance connected between the input and output of an inverting voltage amplifier of gain G.





- Looking from the input, the current going through the feedback element is:

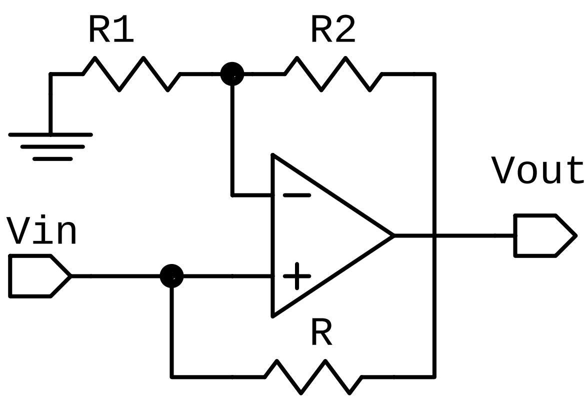
$$i_{in,F} = \left(v_{in} - v_{out}\right)Y = \left(1+G\right)Yv_{in} \Rightarrow Y_{M,in} = \left(1+G\right)Y$$

- Likewise, looking from the output, the amplifier has a gain=-1/G, so the extra current going into the feedback element is:

$$i_{out,F} = \left(1+\frac{1}{G}\right)Yv_{out} \Rightarrow Y_{M,out} = \left(1+\frac{1}{G}\right)Y$$

- These considerations lead to the equivalence of the two diagrams above in terms of their input and output admittance.
- **NOTE: Only if the amplifier is ideal its gain will not change!!!**

Negative Impedance converter



From the Miller Theorem,

$$Z_{in} = \frac{R}{1-G} = \frac{R}{1-\left(1+\frac{R_2}{R_1}\right)} = \frac{-RR_1}{R_2}$$

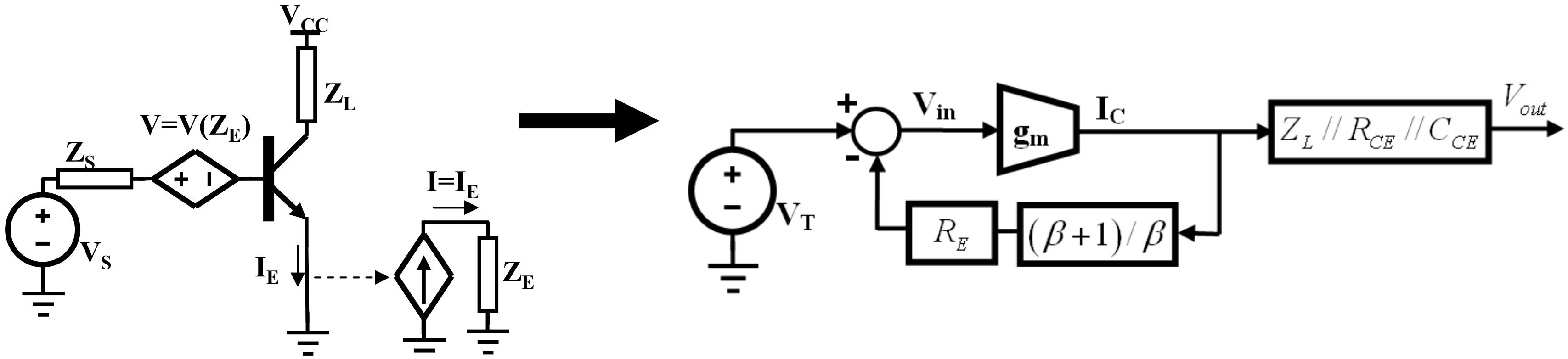
Since the op-amp with R1 and R2 form a voltage amplifier of gain

$$G = 1 + \frac{R_2}{R_1}$$

This method is used to

- *synthesise negative resistances, C's, L's*
- *Invert a given impedance (think of a capacitor in the position of R_1)*
- *Multiply or divide impedance magnitudes (note the ratio R_2/R_1)*

The emitter degenerated CE amplifier: series-series feedback



We can now apply the feedback equations: (the limit is for large transconductance)

$$\frac{V_{out}}{V_T} = \frac{-g_m\left(Z_L // R_{CE} // C_{CE}\right)}{1+g_m R_E\left(\beta+1\right)/\beta} \simeq \frac{-g_m Z_L}{1+g_m R_E} \rightarrow -\frac{Z_L}{R_E}$$

The closed loop amplifier behaves an amplifier with a reduced transconductance

$$g_m' = \frac{g_m}{1+g_m R_E\left(\beta+1\right)/\beta} \simeq \frac{g_m}{1+g_m R_E} \rightarrow \frac{1}{R_E}$$

The input impedance can easily be calculated (note we have included the shunt Miller effect) :

$$Z_{in} = Z_{in0}\left(1+g_m R_E\left(\beta+1\right)/\beta\right) \text{ with } Z_{in0} = \frac{\beta}{g_m} // \left(C_{BE}+\left(A_V+1\right)C_{BC}\right)$$

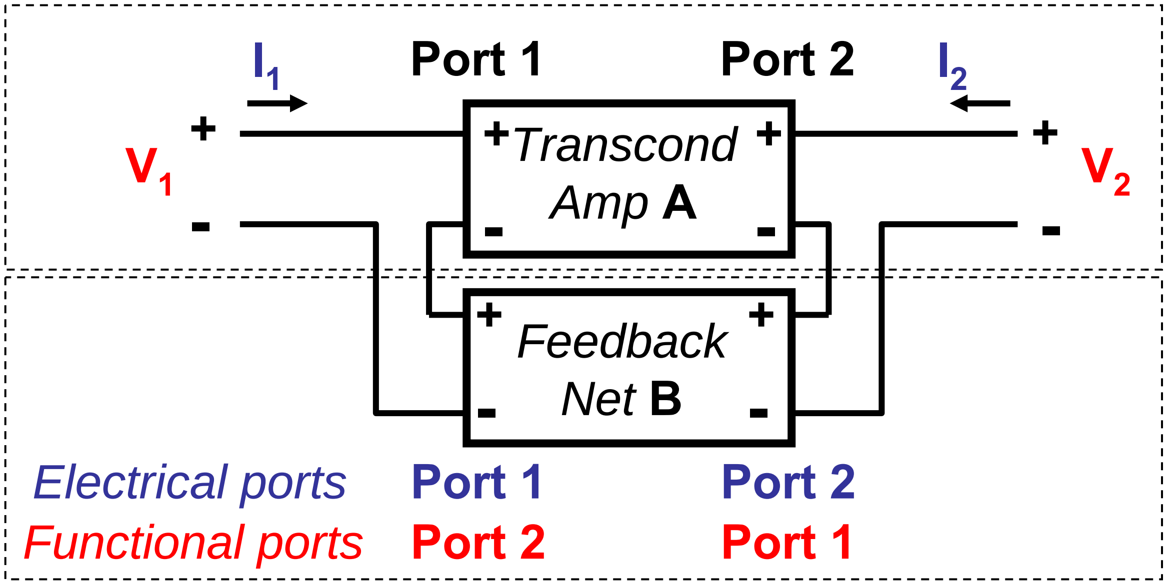
The output impedance is

$$Z_{out} = Z_{out0}\left(1+g_m R_E\left(\beta+1\right)/\beta\right) \simeq R_{CE}\left(1+g_m R_E\right)$$

The frequency response is again calculated from the input and output voltage dividers.

Feedback on transconductance amplifiers

Series - Series connection: Add Z parameters



Function of feedback net:
Measure output I
Correct (mix) input V
i.e. it improves a Y-amp

Shared electrical variables:

Input: I_1 , I_2

Output: V_1 , V_2

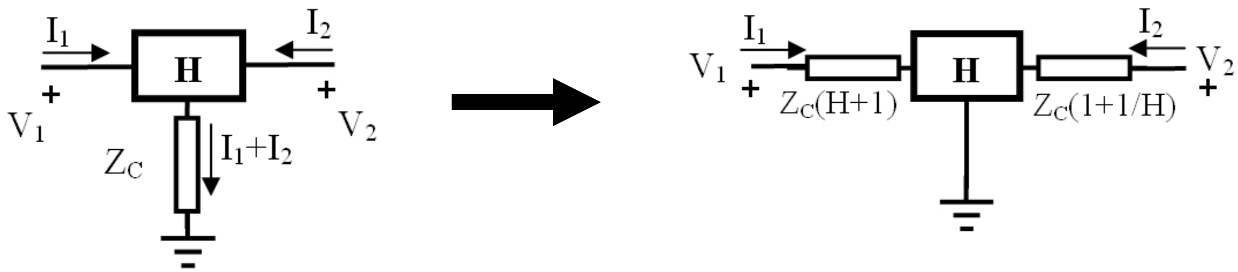
The feedback network is functionally a transimpedance amplifier from Port2 → Port1
Electrically both networks must be treated as transimpedance amplifiers P1→P2
to account for the shared (input) electrical variables.
In the calculation we consider the ELECTRICAL description:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} V_{1A} \\ V_{2A} \end{bmatrix} + \begin{bmatrix} V_{1B} \\ V_{2B} \end{bmatrix} = \mathbf{Z_A} \begin{bmatrix} I_{1A} \\ I_{2A} \end{bmatrix} + \mathbf{Z}_B \begin{bmatrix} I_{1B} \\ I_{2B} \end{bmatrix} = \left(\mathbf{Z_A} + \mathbf{Z_B}\right) \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \mathbf{Z_{A+B}} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

We will study this type of connection when we study transistor amplifiers

The 2nd form of the Miller theorem – series feedback

- ***consider an impedance connected in series with the common terminal of a current amplifier of gain H.***



- ***Looking from the input, the voltage developed on the feedback element is:***

$$V_Z = \left(i_{in} + i_{out}\right) Z = \left(1 + H\right) Z v_{in} \Rightarrow Y_{M,in} = \left(1 + H\right) Z$$

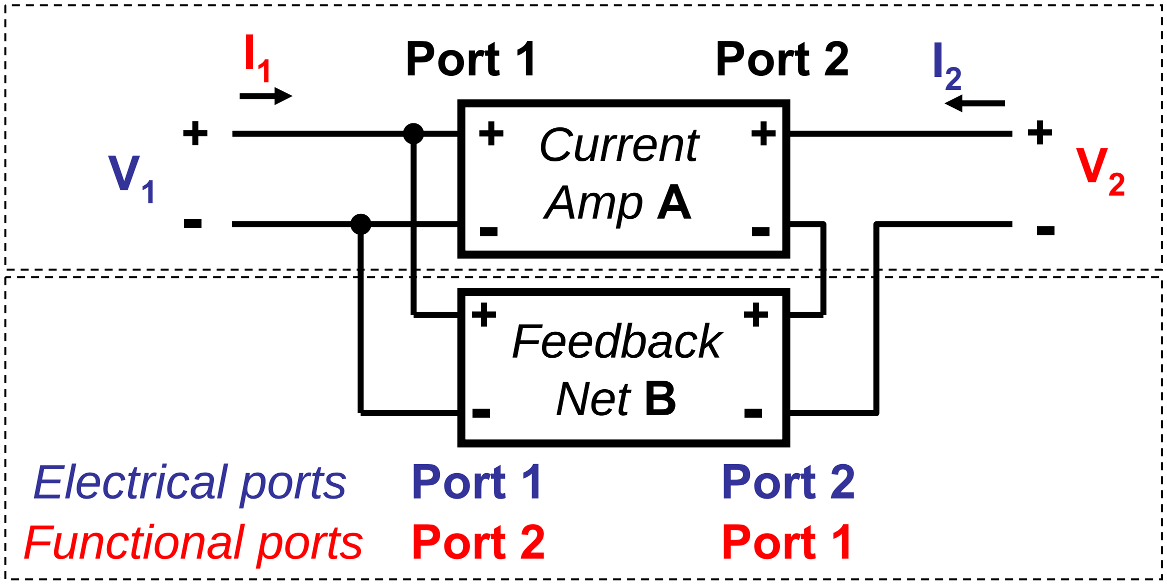
- ***Likewise, looking from the output, the amplifier has a gain=1/H, so the voltage developed on the feedback element is:***

$$V_Z = \left(1 + \frac{1}{H}\right) Z i_{out} \Rightarrow Z_{M,out} = \left(1 + \frac{1}{H}\right) Z$$

- ***These considerations lead to the equivalence of the two diagrams above in terms of their input and output impedance.***
- ***NOTE: Only if the amplifier is ideal its current gain will not change as a result of the series-series feedback.***

Feedback on current amplifiers

The Shunt - Series connection: Add G parameters



Function of feedback net:
Measure output I
Correct (mix) input I
i.e. it improves an H-amp

Shared electrical variables:

Input: V_1 , I_2

Output: I_1 , V_2

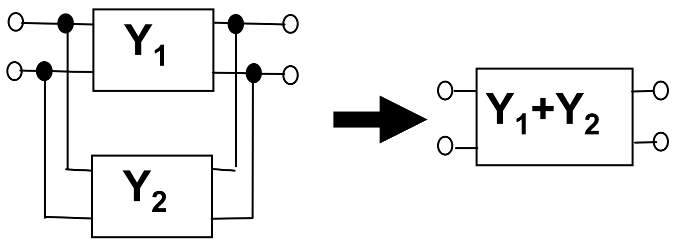
The feedback network is functionally a current amplifier from Port2 → Port1
Electrically both networks must be treated as voltage amplifiers P1 →P2
to account for the shared (input) electrical variables.
In the calculation we consider the ELECTRICAL description:

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_{1A} \\ V_{2A} \end{bmatrix} + \begin{bmatrix} I_{1B} \\ V_{2B} \end{bmatrix} = \mathbf{G_A} \begin{bmatrix} V_{1A} \\ I_{2A} \end{bmatrix} + \mathbf{G_B} \begin{bmatrix} V_{1B} \\ I_{2B} \end{bmatrix} = \left(\mathbf{G_A} + \mathbf{G_B} \right) \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \mathbf{G_{A+B}} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

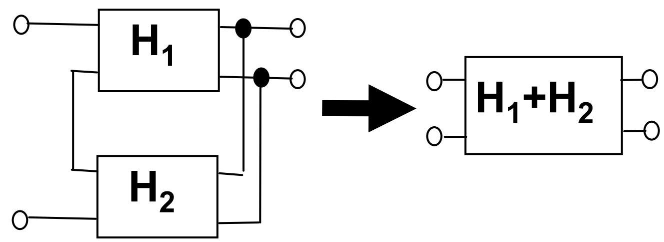
We will study this type of connection when we study transistor amplifiers

2-port network feedback connection rules

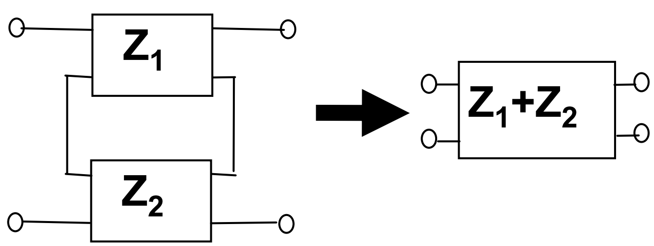
Shunt – Shunt: add Y



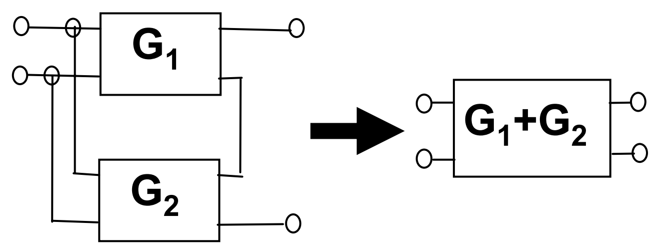
Series – Shunt: add H



Series – Series: add Z



Shunt – Series: add G



Series-shunt feedback:
Effect on input – output impedance

A real op-amp has:

- Finite input impedance
- Finite output impedance
- Finite Gain

Treat the feedback network as if it draws no current. This is equivalent to: $Z_o \ll [R_1, R_2] \ll Z_i$

The input impedance is derived from:

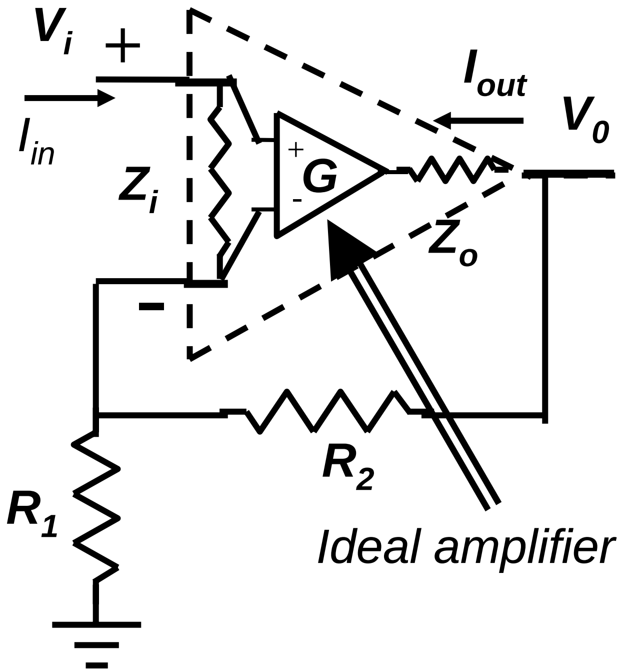
$$(i_{in} Z_i G + i_{out} Z_o) H = v_{in} - i_{in} Z_i \Rightarrow$$

$$i_{in} Z_i (1 + GH) = v_{in} - i_{out} Z_o H \Rightarrow$$

$$Z_{in} = \left.\frac{\partial v_{in}}{\partial i_{in}}\right|_{i_{out}=0} = Z_i (1 + GH)$$

The output impedance is:

$$(v_{in} - v_{out} H) G = v_{out} - i_{out} Z_o \Rightarrow Z_{out} = \left.\frac{\partial v_{out}}{\partial i_{out}}\right|_{v_{in}=0} = \frac{Z_o}{(1 + GH)}$$



Positive feedback

Same analysis as negative feedback, apart for H → -H

Positive feedback an be used to do things negative feedback cannot do:

- Introduce hysteresis (e.g. Schmitt Trigger)
- Generate negative impedances
- Invert an impedance
- Note that under positive feedback we can have F=1-GH=0
- If F=0 we (in theory) can turn an amplifier into an ideal version by a suitable feedback connection and GH=1.
- GH=1, when it occurs at a finite (i.e. non zero) frequency, is the Barkhausen condition for oscillation

The op-amp is called "**operational**" precisely because it can be used to perform mathematical operations

- *on signals (addition, subtraction, integration, differentiation, multiplication by a scalar,…)*
- *on operators (inversion)*
- *on impedances (negation, inversion, multiplication, division,…)*

The dominant pole approximation

•an op-amp has never an infinite gain at DC

• the op-amp gain as a function of frequency is adequately described by:

$$A_v(f) = \frac{A_{DC}}{1+s\tau} = \frac{A_{DC}}{1+jf/f_p} = \frac{A_{DC} f_p}{f_p + jf}$$

Where A_{DC} is the DC gain of the amplifier, typically 10^4 – 10^6

f_p is the **dominant pole frequency,** typically 10 Hz.

The product $A_{DC}f_p$ is called the **gain-bandwidth product (GBW).**

The GBW is a characteristic constant of the op-amp, typically 10^6-10^8

When we do AC analysis we **must consider** the finite complex gain of

The amplifier, especially when we try to get high gain at high frequencies.

Invariance of the gain – bandwidth product

Consider a non-inverting amplifier, and a dominant pole op-amp
Applying the feedback theory we get the **closed loop gain:**

$$A_V = \frac{\dfrac{A_{DC} f_p}{f_p + jf}}{1 + \dfrac{A_{DC} f_p}{f_p + jf} H} = \frac{A_{DC} f_p}{f_p + jf + A_{DC} f_p H} = \frac{GBW}{f_p + jf + GBW \cdot H}$$

The DC gain is:

$$A_V\left(f = 0\right) = \frac{GBW}{f_p + GBW \cdot H}$$

The pole of this amplifier is at: $f_0 = f_p + GBW \cdot H$

It follows that the product of the DC gain of a non-inverting amp and its pole position equals the gain bandwidth product of the op-amp!
This is only true for a dominant pole op-amp! (i.e. most voltage mode amplifiers)

Conclusions

- Feedback reduces gain
- Feedback reduces component and environmental sensitivity
- Feedback increases linearity
- There are 4 ways to apply electronic feedback
- Feedback can be used to modify input and output impedances:
  - *A series negative feedback increases impedance*
  - *A series positive feedback decreases impedance*
  - *A shunt negative feedback increases admittance*
  - *A shunt positive feedback decreases or zeroes admittance*
- Positive feedback can lead to dynamic instability
- Op-amps are modelled as dominant pole systems
  - *Op-amps have finite DC gain*
  - *Op-amps have a low frequency pole.*
  - *Amplifiers built with op-amp are subject to a constant GBW*

2^{nd} order filter transfer functions: Review

Second order filter transfer functions are all of the following form:

$$H(s) = H_0 \frac{C(s/\omega_0)^2 + 2B\zeta s/\omega_0 + A}{(s/\omega_0)^2 + 2\zeta s/\omega_0 + 1} \;,\quad Q = \frac{1}{2\zeta}$$

H_0 is the overall amplitude, ω_0 the break (or peak) frequency, and ζ the damping factor

ζ isrelated to the quality factor Q by: $Q=1/2\zeta$

The 3dB bandwidth of an underdamped 2^{nd} order filter is approx 1/Q times the peak frequency.

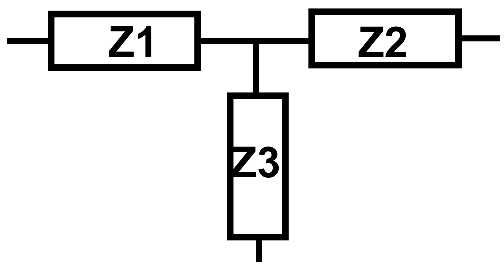
The coefficients A, B, C determine the function of the filter:

Function	A	B	C
Low Pass	1	0	0
High Pass	0	0	1
Band Pass	0	1	0
Band Stop	1	0	1
All Pass	1	-1	1

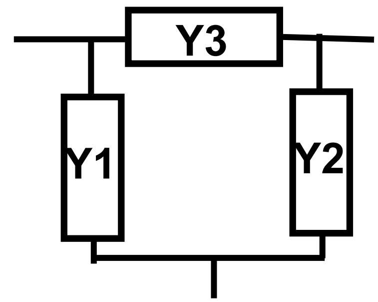
Tee – Pi transformations

When analysing active band pass or band stop active filters we often encounter the "twin-tee" passive notch filter topology

This requires quite a bit of algebra to compute, so we prove a, useful for simplifying networks, lemma:



is equivalent to



Proof: write the z matrix of the Tee< and the y matrix of the Pi and require that the two circuits are representations of same network:

$$\mathbf{Z}_{Tee} = \begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix}, \mathbf{Y}_{Pi} = \begin{bmatrix} Y_1 + Y_3 & -Y_3 \\ -Y_3 & Y_2 + Y_3 \end{bmatrix}, \mathbf{Y}_{Pi} = \mathbf{Z}_{Tee}^{-1} \Rightarrow$$

$$\Rightarrow \begin{cases} Z_1 = \dfrac{Y_2}{Y_1Y_2 + Y_1Y_3 + Y_2Y_3}, Z_2 = \dfrac{Y_1}{Y_1Y_2 + Y_1Y_3 + Y_2Y_3}, Z_3 = \dfrac{Y_3}{Y_1Y_2 + Y_1Y_3 + Y_2Y_3} \\ Y_1 = \dfrac{Z_2}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}, Y_2 = \dfrac{Z_1}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}, Y_3 = \dfrac{Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3} \end{cases}$$

Higher order filter synthesis using 2nd order sections

- **A general filter transfer function is of the form:**

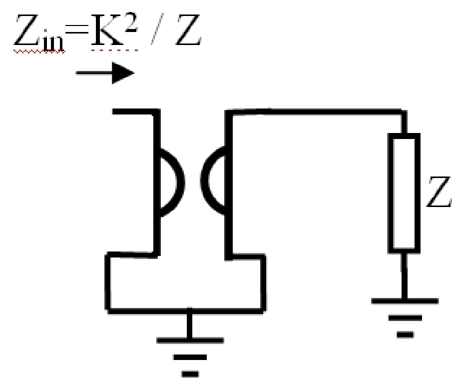
$$H(s)=\frac{P_n(s)}{Q_m(s)}=\frac{\sum_{i=0}^{n}a_k x^k}{\sum_{i=0}^{m}b_k x^k}=\frac{(s-z_0)(s-z_1)\cdots(s-z_n)}{(s-p_0)(s-p_1)\cdots(s-p_n)}$$

- **P(s) and Q(s) have real coefficients. To make a higher order filter:**
  - ***factor P(s) and Q(s) into quadratic and linear factors***
  - ***Implement factors as biquads***
  - ***Cascade biquad sections to obtain the original transfer function***
  - ***Note that the roots of P, Q are real or come in conjugate pairs.***
- **The centre frequencies and damping factors of the sections required to implement standard forms (Butterworth, Chebyshev, Elliptic etc) are tabulated. Tables are included in CAD software for automated synthesis**

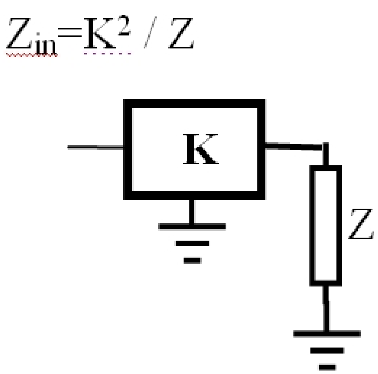
A useful network transformation:
Impedance inversion and the gyrator

A gyrator can perform

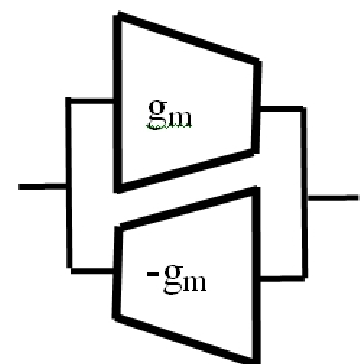
- impedance inversion (L←→C)
- Impedance scaling
- series – parallel connection conversion!



"Proper" symbol of gyrator



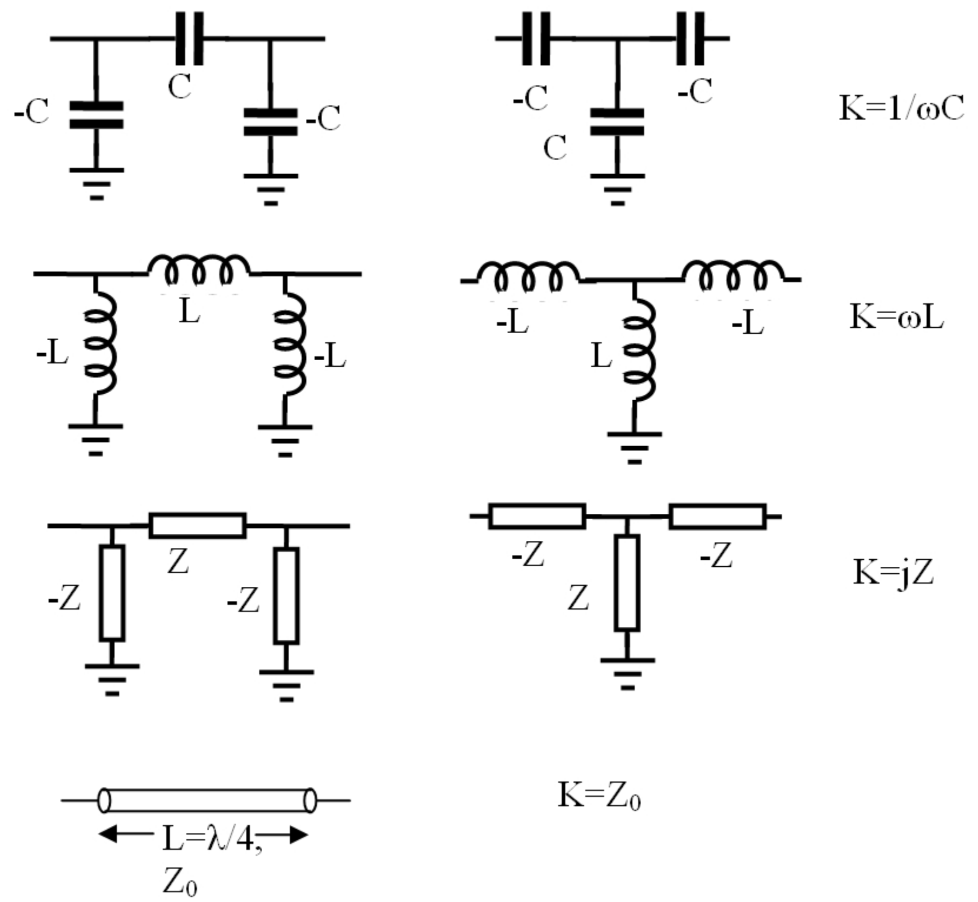
Alternate symbol



Simple active implementation (very popular by analogue CMOS designers. Each gm is made of a MOSFET or two!)

*Passive Gyrators

- ¼ wavelength transmission line
- Pi and Tee networks with negative elements



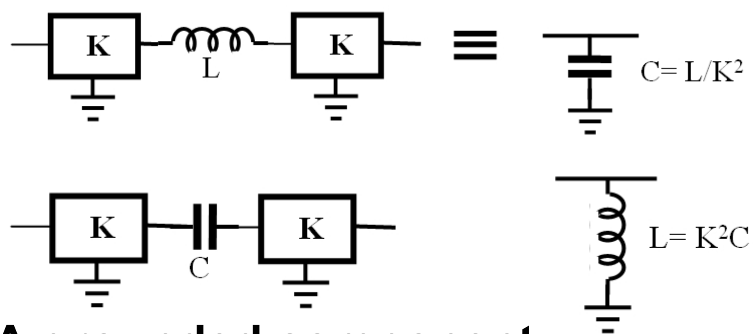
negative values of components will be added to preceding and subsequent stage impedances resulting in overall positive impedances! Note that for narrowband signals, eg, –L is a capacitor!

Ladder LC filters can be synthesised only with capacitors and gyrators

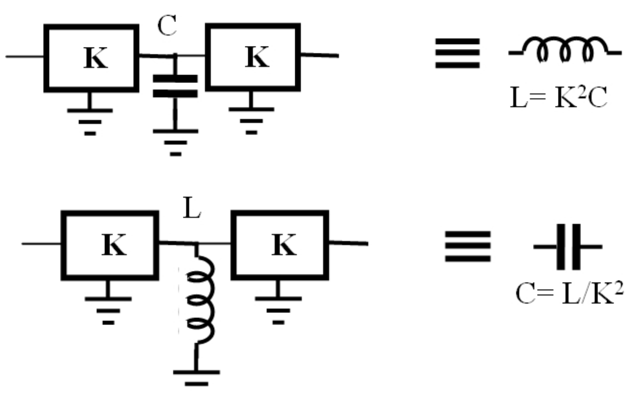
Z, -Z is completely arbitrary, can be a filter transfer function and more…

Gyrator function - basics

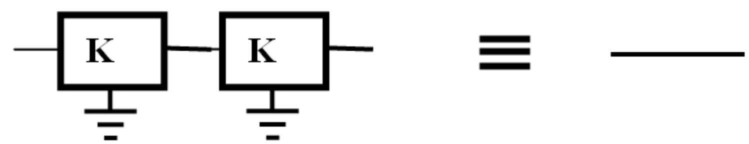
- A series (floating) component between two gyrators appears inverted and grounded



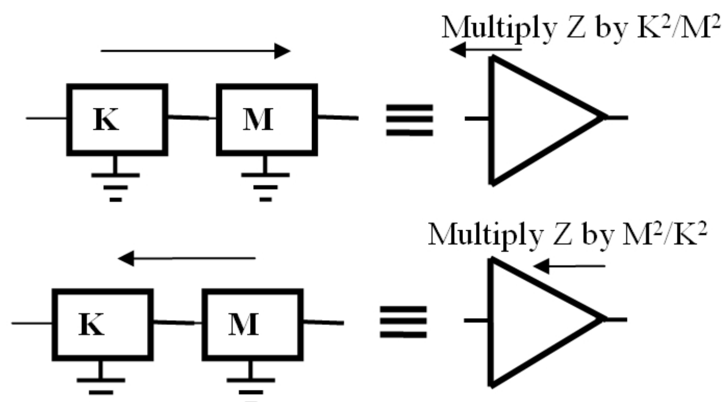
- A grounded component between two gyrators appears inverted and in series



•Two identical gyrators in series are the identity operator



•Two different gyrators in series perform direction sensitive impedance multiplication by a constant:



Some more gyrator identities

or, how to make e.g. a series resonance circuit when you only have parallel resonators in your component box… and vice versa