1 Radio and Radar Links

1.1 Objectives

- Become familiar with some of common antenna and radio link terminology.
- Perform calculations involving antenna gain, aperture, directivity, efficiency and noise temperature.
- Calculate the power budget of telemetry and radar links.

1.2 Radio links

1.2.1 Frequency Bands

What we will discuss this term occurs at high frequencies. We usually state the frequency of operation explicitly. However, in microwave engineering there is a custom of talking about frequency Bands, with coded names. The codes can be confusing especially given that there are several schemes in use. Indeed frequency codes were meant to be confusing since high frequency engineering has a military origin. One of the common schemes in use adopted by the IEEE as a standard, is summarised in table 1.

Frequency (GHz)	Designation	$\lambda(cm)$
< 0.1	VHF	>300
.1-1	UHF	300-30
1-2	L	30-15
2-4	S	15-7.5
4-8	C	7.5-3.75
8-12.4	X	3.75-2.5
12.4-18	Ku	≃2
18-26.5	K	≃1
26.5-40	Ka	1.1-0.6
50-75	V	0.6-0.4
75-110	W	.4-.3
100-1000	mm waves	3-0.3 mm
> 1000	Far IR	<200μm

Table 1: The IEEE Frequency band designations

1.2.2 Power budget

We will examine an abstracted radio link, consisting of a transmitter, a receiver and their antennas (fig. 1). For the moment we will not be concerned with any detailed modelling

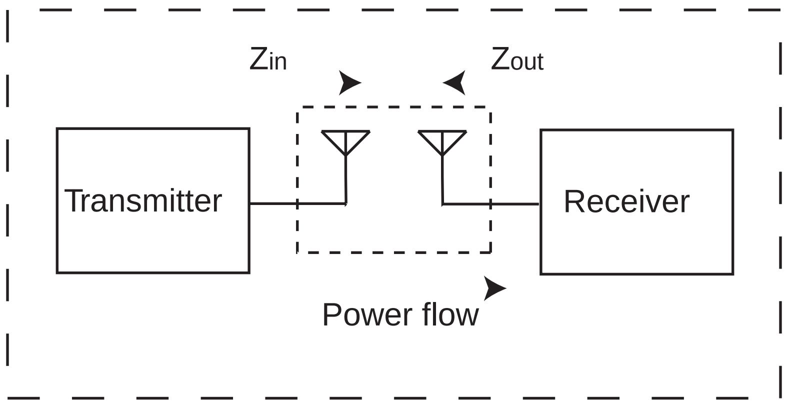

Figure 1: An abstracted radio link. The pair of antennas can be considered as a signal processing block with well defined input and output impedance, and power gain.

of the transmitter and receiver, other than the obvious necessity to maintain energy flow between the two, in order to transmit information. We will not be concerned with the operation of the antennas either, and will effectively treat them as interconnect cables. To see how this can be true, if we imagine both antennas inside an opaque box, we cannot really know if transmitter and receiver are connected by the antenna pair, or by a cable and a filter, and optical beam, or a transformer!
The power P_{rec} received by the receiver antenna, when power P_{trans} was transmitted by the transmitting antenna is given by the Friis Formula:

$$P_{rec} = P_{trans} \frac{A_{eff,T} A_{eff,R}}{\lambda^2 r^2} \tag{1}$$

Where:
$A_{eff,T}$: The effective transmitter area
$A_{eff,R}$: The effective receiver area
λ : The free space wavelength for the frequency at which we operate the link
r : The distance between transmitter and receiver.
We can rearrange this equation to read:

$$P_{rec} = P_{Trans} \frac{A_{eff,T}}{\lambda^2} \frac{A_{eff,R}}{\lambda^2} \frac{\lambda^2}{r^2} \tag{2}$$

Effectively, this formula states that the apparent power gain of the link is the product of the antenna areas over the square of the distance, all lengths measured in wavelengths!
We turn now to defining a few basic antenna terms, and to getting a feel about the plausibility of the Friis formula.

1.3 Antenna Definition and properties

An antenna is a transducer. A transmitting antenna converts the guided waves travelling on the cables connecting it to an electronic transmitter to travelling waves in space. The receiving antenna interacts with the electromagnetic fields in its vicinity, and "collects" energy to convert it into guided waves on the cable linking it with a receiver. Antennas are **reciprocal** devices. We may reverse the roles of transmitting and receiving antennas in a radio link without changing the **gain** of the link. Reciprocity allows us to chose whether we regard a particular antenna as a transmitting or a receiving device when we calculate its properties. It further allows us to experimentally characterise an antenna in whichever of the two roles is more convenient.

At radio frequencies am antenna is usually a metal structure, such as a wire, a rod, a helix, a dish. We can form one and two dimensional arrays of elementary antennas in order to increase or even steer directivity. At higher frequencies, such as millimetre waves, far infrared, visible light, and near-UV we can have dielectric structures as antennas (e.g. a glass lens is an antenna!). In principle an antenna works because Maxwell's equations impose boundary conditions between the currents and voltages on the antenna structure and the electromagnetic field around it. For all practical purposes the circuit engineer can regard an antenna as yet another circuit element, with well defined terminal characteristics.

The electromagnetic field around an antenna is divided in the near and far field regions. At distances relatively near the antenna we say we are in the **near field** region. There we have interaction between the charges and currents on the antenna itself and the fields. This interaction results to both energy flow and energy storage. The imaginary part of the antenna impedance is due to energy storage in the near field region. Thus an antenna may appear inductive if energy is predominantly stored in the magnetic field, or capacitive, if energy is predominantly stored in the electric component of the near field. Far enough away from the antenna, we say we are in the **Far Field**, or the **Radiation region,** where we effectively have radial wave propagating towards, or away from, the antenna. In the radiation region electric and magnetic fields are transverse to the direction of propagation, i.e. the wave is a transverse wave. The division between the near field and the far field is arbitrarily set to:

$$R_r = \frac{2L^2}{\lambda} \tag{3}$$

Where L is the physical size of the antenna, such as its aperture if we are talking about a dish or a slot, or its length, if we are talking about a rod. R_r is the distance from which the antenna effectively appears as a point source.

In the far field the wave can be decomposed to a superposition of two polarised waves. In practice we almost always assume to be in the radiation region. Reciprocity allows us to neglect the field distortion resulting from placing the antenna inside the field.

An antenna can be phenomenologically characterised by a number of properties which we review below:

1.3.1 Radiation resistance

If we apply an AC voltage to the terminals of a transmitting antenna, we know (by experience) that it will radiate. Conservation of energy dictates we must supply the power radiated. Consequently, if we apply a voltage V (RMS) to the terminals of an antenna, it will appear to have a finite impedance. Since energy will appear to be dissipated by the antenna, the impedance has a positive real part R, a "Resistance", which is called the **Radiation Resistance** of the antenna:

$$P = \frac{V^2}{R} \tag{4}$$

To see why an antenna radiates, and how the radiation resistance comes about, we calculate the radiation resistance of a very short ($\lambda >> l$) dipole.
An accelerating charge q, radiates a total power P:

$$P = \frac{\mu^2 \dot{v}^2 q^2}{6\pi Z} \tag{5}$$

where
μ is the magnetic permeability of the medium, $\mu = \mu_0 = 4\pi \times 10^{-7} Hm^{-1}$ in vacuum.
ϵ is the dielectric constant, $\epsilon = \epsilon_0 = 8.854 \times 10^{-12} Fm^{-1} in$ vacuum.
$c = \frac{1}{\sqrt{\mu\epsilon}}$ is the speed of light in the medium, $c = c_0 = 3 \times 10^8 m\,\mathrm{sec}^{-1}$ in vacuum, and
$Z = \sqrt{\frac{\mu}{\epsilon}}$ is the "natural" impedance of the medium. $Z = Z_0 = 377\ \Omega$ in vacuum.
If we have a linear charge distribution, of extent x , then the continuity between charge and current means that:

$$x\dot{I} = q\dot{v} \tag{6}$$

and, therefore,

$$P = \frac{\mu^2 \dot{I}^2 x^2}{6\pi Z} \tag{7}$$

the power radiated by a short ($x < \lambda/10$) dipole, where the spatial average of the current is approximately 1/2 the terminal excitation current, is:

$$P \simeq \frac{\pi Z_0}{6}\left(\frac{x}{\lambda}\right)^2 I_0^2 = 197\Omega \cdot \left(\frac{x}{\lambda}\right)^2 I_0^2 \tag{8}$$

Where I_0 is the (time average) amplitude of the current driving the antenna. Comparing this result with the familiar expression for the dissipation on a resistor driven by a current amplitude I_0 and real. Therefore, the antenna presents a *radiation resistance* of

$$R = 197\Omega \cdot \left(\frac{x}{\lambda}\right)^2 ohms \tag{9}$$

The radiation resistance is just the real part of the antenna terminal impedance which is almost always complex.

The imaginary part of the antenna impedance can be:

- $X_a = \mathrm{Im}(Z_a) < 0$ (Capacitive)
  if the physical dimension x of the antenna is: $n\frac{\lambda}{2} < x < \frac{\lambda}{4} + n\frac{\lambda}{2}$
- $X_a = \mathrm{Im}(Z_a) > 0$ (Inductive)
  if the physical dimension x of the antenna is: $\frac{\lambda}{4} + n\frac{\lambda}{2} < x < (n+1)\frac{\lambda}{2}$

Note that for "centre fed antennas" we actually talk about *double* the dimension defined above.
If the antenna radiation impedance is $Z = R_r + jX_a$ and its ohmic resistance is r_L, then the power transferred to the antenna from a generator of voltage V with an output impedance $R_G + jX_G$ is:

$$P = \frac{V^2}{4R_G} \frac{4R_G R_r}{(Rr + R_G + r_L)^2 + (X + X_G)^2} = P_{avail} M \tag{10}$$

Where we have also defined the *Mismatch Factor* M which is the fraction of the *Available Power* R_{avail} that reaches the antenna.

$$P_{avail} = \frac{V^2}{4R_G} \tag{11}$$

and

$$M = \frac{4R_G R_r}{(Rr + R_G + r_L)^2 + (X + X_G)^2} \tag{12}$$

The mismatch factor, written differently, recurs in microwave electronics. Keeping it as close to $M = 1$ is the main concern of an RF designer. Note that the available power is the most we can hope to transmit from a generator to a load under *Conjugate Matching,* i.e. when:

$$Z_L = Z_G^* \tag{13}$$

This simple example calculation of the radiation resistance of a small dipole antenna neglects interference of the field components due to currents in different physical parts of the antenna. To calculate the radiation resistance of a bigger antenna we would need to evaluate the total field and from this the total power flux. Such a calculation is analytically feasible only for the simplest geometries. Two common antennas are the half wave dipole, which has a radiation resistance of 73.13 ohms, and the folded dipole (the common "T" shaped FM antenna) with R=292.5 ohms.

1.3.2 Directivity, gain and beam size

No antenna radiates isotropically. Nonetheless, the field intensity will always vary inversely with the distance from the antenna
inside the radiation zone. This happens because the total energy flux cannot depend on distance.
An isotropically radiating antenna would have a power flux density independent of the polar direction:

$$S = P_{trans}\frac{1}{4\pi r^2} \tag{14}$$

We can define the energy flux per unit solid angle:

$$U(\vartheta, \varphi) = \frac{\partial U}{\partial \Omega} = r^2 \frac{\partial U}{\partial A} \tag{15}$$

$U(\vartheta, \varphi)$ is a function of the (spherical) polar coordinates θ and ϕ to the direction of propagation, independent of distance. The isotropic source would have a flux density of:

$$U_{isotropic} = \frac{P_{trans}}{4\pi} \tag{16}$$

The **Directivity** D of an antenna is defined in terms of $U(\vartheta, \varphi)$ as:

$$D(\vartheta, \varphi) = \frac{U(\vartheta, \varphi)}{U_{avg}} \tag{17}$$

What is usually quoted is the maximum directivity, or simply the Directivity of the antenna:

$$D = \frac{U_{max}}{U_{avg}} \tag{18}$$

Equally often, reference is made to the *Gain* of the antenna:

$$G = kD \tag{19}$$

Where k the electrical efficiency of the antenna, the ratio of radiated to input power.
We can estimate the directivity of an antenna by considering the angular size if the beam. A directive beam will have several lobes. The main lobe will be the desirable part of the antenna far field. Assume first that the intensity of the main beam far exceeds the intensity of all the other lobes. Then, almost all the radiated power is in the main lobe. We consider the *beam solid angle*, also called the *beam area* :

$$\Omega_M = \int\int_{mainlobe} \frac{U(\theta, \phi)}{U_{max}} d\Omega \tag{20}$$

with $d\Omega = \sin(\theta) d\theta d\phi$. Ω_M is approximately equal to the solid angle defined by half the maximum beam intensity around the maximum. The directivity of a beam of area Ω_M is:

$$D \simeq \frac{4\pi}{\Omega_M} \tag{21}$$

Example The beam size of a beam with $U(\theta,\phi) = U_m \cos(\theta)$, U_m is the maximum beam intensity) radiating only in the upper half space ($\theta < \pi/2$) is:

$$\Omega = \int_0^{2\pi} \int_0^{\pi/3} \sin(\theta) d\theta d\phi = \pi \tag{22}$$

The limit of the θ integration was taken to be $\pi/3$ since $\cos(\pi/3) = \frac{1}{2}$. The directivity of this antenna is, therefore, $D = \frac{4\pi}{\Omega} = \frac{4\pi}{\pi} = 4$.
Of special interest, and far easier to calculate, is the directivity of the so-called pencil beam antennas, of half power angular extent θ_{HP} and ϕ_{HP} in the θ and ϕ directions respectively. To the lowest approximation:

$$D = \frac{41000}{\theta_{HP}\ \phi_{HP}} \tag{23}$$

Since 4π steradians is approximately 41000 square degrees.
Typical pencil beam antennas are large dishes, apertures, helicals and arrays. It is interesting to note that the *resolution* of the antenna equals its beam size, and also that the beam width at half power is almost 1/2 the width at the first zeroes.
Example The directivity of a 2^o pencil beam is

$$D = \frac{41000}{4} \approx 10000 = 40dB \tag{24}$$

conversely, the main lobe beam size of a lossless ($k = 1$)antenna with gain G=60dB is approximately:

$$\theta_{HP} = \phi_{HP} \approx \sqrt{\frac{41000}{10^6}} \approx 0.2^o = 12' \tag{25}$$

In noise calculations the beam efficiency, defined in terms of the main and minor lobe areas, Ω_M and Ω_m respectively, is important:

$$\epsilon_M = \frac{\Omega_M}{\Omega_M + \Omega_m} \tag{26}$$

Note that the denominator above does not equal 4π.
We may wish to define the *effective Aperture* of an antenna, A_{eff}, in terms of the gain of the antenna:

$$A_{eff} = \frac{G\lambda^2}{4\pi} \tag{27}$$

It turns out A_{eff} is nearly equal to the physical size of the antenna. Indeed, we can define the *aperture efficiency* as:

$$\eta_a = \frac{A_{eff}}{A_{physical}} \tag{28}$$

Consequently, this equation allows one to estimate the gain of a given size antenna, or, conversely, to estimate the necessary size of an antenna, after a system calculation has indicated a particular gain requirement.

The effective aperture of the antenna is the "collecting area" it presents when it is used as a receiver, and in the case of apertures and dishes it is about 50% - 60% the physical area. Monopoles, dipoles and other wire antennas have **non-zero** apertures, as listed in table 2.
Example: The previous 2^o pencil beam antenna has at $\lambda = 21cm$ an effective area of

$$A_{eff} = \frac{1}{4\pi}(0.21)^2 \times 10^4 = 35m^2 \tag{29}$$

implying a physical area about twice as large, i.e. 70 sq. m, a linear dimension of the order of 10 m for a circular dish.
Some engineers use the term "loss aperture" A_L i.e. the antenna area increment which would transmit or receive the power lost in ohmic heating on the antenna structure. The loss aperture is given by:

$$\frac{A_L}{A_e} = \frac{r_L}{R_r} \tag{30}$$

Clearly a good antenna must have a much smaller loss aperture than effective aperture, or equivalently, must have a large radiation resistance compared to its ohmic resistance. Ultimately, the efficiency, k, of an antenna will be:

$$k = 1 - \frac{r_L}{R_r} \tag{31}$$

We may now wish to use the definition of the antenna gain to rewrite the Friis transmission formula as:

$$P_{rec} = \left(\frac{1}{4\pi}\right)^2 P_{trans} G_T G_R \left(\frac{\lambda}{r}\right)^2 \tag{32}$$

A useful application of the concept of gain, and the simplifications it affords is Radar (problem 4). In radar operation it is the same antenna acting as a transmitter and receiver, and the target re-broadcasts isotropically (i.e. with gain=1) all the power it receives. Then, the radar equation , giving the received radar signal is: (P_T power incident on target, P_{XR} Power received by the radar from the target, G_X the target "Gain")

$$\begin{aligned} P_T &= \left(\frac{1}{4\pi}\right)^2 P_X G_X G_T \left(\frac{\lambda}{r}\right)^2 \\ P_{XR} &= \left(\frac{1}{4\pi}\right)^2 P_T G_X G_T \left(\frac{\lambda}{r}\right)^2 = \ldots = P_X \frac{A_X^2 A_T^2}{\lambda^4 r^4} \end{aligned} \tag{33}$$

We can now see why short wavelengths are essential for radar, as they are for long range communications. We can also understand why "edgy" shapes (which can present much smaller effective crosssection) or absorbent coatings of low conductivity are used to reduce the radar signature of objects.

Description	R_e	Aperture	h_e	**D**	**D**
	Ω	λ^2	meters		dB
Isotropic		$1/4\pi$=0.079		1	0
Short dipole, $l < \lambda/10$	$80\left(\frac{\pi l I_{avg}}{\lambda I_0}\right)^2$	$\frac{3}{8\pi} = 0.119$	$\frac{l I_{avg}}{I_0}$	1.5	1.76
Short dipole, $l = \lambda/10$	7.9	0.119	$\lambda/10$	1.5	1.76
$\lambda/2$ dipole	73	0.13	λ/π	1.64	2.15
Small loop, n turns Any shape Area $A < \lambda^2/100$	$n^2 31200\left(\frac{A}{\lambda^2}\right)^2$	0.119	$2\pi n\frac{A}{\lambda}$	1.5	1.76

Table 2: Parameters of some wire antennas

1.3.3 Antenna Height

If we know that the electric field radiated has an intensity E (Volts/meter) in the vicinity of the antenna then the antenna receiving this field will develop at its terminals a voltage V. The effective height h of the antenna is defined thus:

$$h_e = \frac{V}{E} = \sqrt{\frac{2R_r A_{eff}}{Z_0}} \tag{34}$$

This relationship easily extracted, apart of numerical factors from conservation of energy. Note that in terms of the amplitude of the electric and magnetic field, the energy flux (Poynting vector) is given by:

$$\vec{\mathbf{S}} = \frac{1}{2}\vec{\mathbf{E}} \times \vec{\mathbf{H}} \Rightarrow S = \frac{E^2}{2Z_0} = \frac{H^2 Z_0}{2} \tag{35}$$

It follows that

$$Z_0 = \frac{E}{H} = \frac{1}{\epsilon c} = \sqrt{\frac{\mu}{\epsilon}} \tag{36}$$

As seen in table 2, the effective height of a wire antenna is not its length, but rather an average of its length over the current distribution on the antenna.
Example The antenna height of a $1m^2$ dish with a 50% aperture efficiency, and a 75 ohm radiation resistance detector is:

$$h_e = \sqrt{\frac{2\ 75\ 0.5}{377}} = 0.198m \tag{37}$$

The example indicates that in planar antennas the height in general differs from the physical dimension.

1.4 Arrays of antennas

To attain high gain, we may use arrays of antennas. A general form for the pattern of a single antenna is:

	Directivity	Beam Width
Linear broadside array	$2L$	$\frac{50.8^o}{L}360^o$
Linear end fire array	$2\pi L$	
Square broadside aperture	$4\pi L^2$	$\left(\frac{50.8^o}{L}\right)^2$
Circular broadside aperture	$\pi^2 L^2$	$\left(\frac{58^o}{L}\right)^2$

Table 3: Directivities and beam widths (in square degrees) of some apertures and arrays. L is the dimension or diameter in wavelengths.

$$E(\mathbf{r}) = f(\vartheta, \phi)\frac{e^{-jk_0 r}}{4\pi r} \tag{38}$$

The total electric field due to the array, each element located at a position x_i with respect to the array centroid, and being driven with a relative amplitude C_i and phase a_i is the sum of the individual patterns:

$$E(\mathbf{r}) = f(\vartheta, \phi)\frac{e^{-jk_0 r}}{4\pi r}\sum_{i=1}^{N} C_i e^{ja_i - jk_0 \widehat{\mathbf{r}}\cdot\mathbf{x}_i} \tag{39}$$

In this expression we assume that the variation of the distance of each array element only affects the phase, and its effect on magnitude is insignificant.
We note that the pattern is the product of the element pattern and the array pattern. The directivity is also proportional to the product of the element directivity and an array function:

$$D \propto |f(\vartheta, \phi)|^2 \left|\sum_{i=1}^{N} C_i e^{ja_i - jk_0 \widehat{\mathbf{r}}\cdot\mathbf{x}_i}\right|^2 \tag{40}$$

Please note that the argument may be iterated, and we may form arrays of arrays, etc. By proper choice of the amplitudes and phases driving the various antenna elements almost any beam shape can be defined, and the beam can be steered in space. Conversely, an array can be used for direction finding, by properly synthesising the beam pattern of a receiving array.
Example: An array of N equidistant point sources, spaced a distance $l_i = x\lambda$apart, and driven uniformely, in phase, has an array pattern of:

$$A(\theta, \phi) \propto \sum_{n=1}^{N} C_i e^{ja_i - jk_0 \widehat{\mathbf{r}}\cdot\mathbf{x}_i} = \sum_{n=1}^{N} e^{j2\pi n x \sin(\vartheta)} = \frac{1 - e^{-j2\pi N x \sin(\vartheta)}}{1 - e^{-j2\pi x \sin(\vartheta)}} \tag{41}$$

which we can simplify to:

$$\begin{aligned} A(\theta,\phi) &\propto e^{-j\pi(N-1)x\sin(\vartheta)}\frac{\sin(\pi Nx\sin(\vartheta))}{\sin(\pi x\sin(\vartheta))} \Rightarrow \\ |A(\theta,\phi)| &\propto \left|\frac{\sin(\pi Nx\sin(\vartheta))}{\sin(\pi x\sin(\vartheta))}\right| \end{aligned} \tag{42}$$

1.5 Antenna noise and system sensitivity

An antenna with radiation resistance R_r placed inside an anechoic cavity at a temperature T, will have the same noise as a resistance R at the same temperature. It will, that is, behave like a noise power source with an available power given by the Nyquist relation (B is the observation bandwidth):

$$P = kTB$$

k is the Boltzmann constant, $k = 1.38 \times 10^{-23}$ J, and T the absolute temperature. This number is 174 dBm/Hz (dBm is dB referred to 1 milliwatt). An antenna looking a sky of temperature T will generate the same noise power density as a resistor R_r at temperature T. To calculate the contribution of an antenna to the system noise, we have to add the noise contributions of all the lobes, each contributing the noise density consistent with the temperature of the object it looks at.
Example: An antenna to be used for a satellite link has a beam efficiency of 0.6. Of the minor lobes 30% are forward looking, and the rest is backward. Assuming the sky temperature is 10 K and the Earth at 300 K, what is the antenna temperature?

$$T_{ant} = 0.6 \times 10 + 0.4 \times 0.3 \times 10 + 0.4 \times 0.7 \times 300 = 6 + 1.2 + 84 = 91.2K$$

the example clearly demonstrates that we cannot neglect the minor lobes or the antenna efficiency in noise calculations.
Of course, what is more interesting is the system noise (or noise figure). A system, in an ambient temperature T_0 consists of:

1. Antenna of noise temperature T_a, and efficiency k_a ($1 - k_a$ represents ohmic losses on the antenna.)
2. Interconnecting transmission line or cable, with transmission coefficient k_c
3. The receiver, of noise temperature T_R, consisting of a number of stages each with noise temperature T_i and gain G_i.

The noise temperature of this system is:

$$T_s = T_a + \left(\frac{1}{k_a} - 1\right) T_0 + \left(\frac{1}{k_c} - 1\right) T_c + \frac{1}{k_c} T_R \tag{43}$$

Since the noise temperature is related to the noise figure by:

$$N = \frac{T_N + T_0}{T_0} \tag{44}$$

It follows that the noise temperature of the receiver is

$$T_R = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \ldots \tag{45}$$

1.6 Exercises

1. Calculate the directivity of an antenna with a raised cosine radiation pattern:

   $$P(\theta, \phi) = P_0 cos^2(\theta)$$

   What is the directivity of a sine radiation pattern? Why are these two numbers different? Draw diagrams of the radiation intensity patterns.

2. Two satellites have on board $1m^2$ antennas to communicate to each other at 10 GHz. They are 10^8 meters apart. assuming 50% aperture efficiency, calculate the minimum transmitter power so that 1pW can be received.

3. Repeat the previous exercise for an antenna temperature of 50^o . The sensitivity condition is now that $S/N \geq 1$. What changes if the receiver has a noise figure F=2dB?

4. (from last year's final exam) On the 10th and 12th of February, 1959 the first attempt was made to bounce a radar beam off the planet Venus. The attempt was unsuccessful due to marginally insufficient signal, i.e. $S/N = 1$. The equipment used by the MIT Lincoln Labs team was:

   - A 265 kW transmitter at a frequency of 440 MHz.
   - The Millstone Hill 26 m diameter parabolic antenna, with an aperture efficiency of 50% and a noise temperature of 20K. This antenna was used for both transmission and reception.
   - a maser preamplifier of which you will calculate the noise temperature. The effective (including the effects of signal averaging) reception bandwidth was $0.65Hz$. It was, a year later, determined by a team at the Jet Propulsion laboratory that the distance of Venus from earth was 68 million kilometres, and that Venus has a radar cross section equal to a 2000 km disk (see note).

   (a) Derive an equation for the signal power received by the radar in terms of antenna dimensions, aperture efficiency, target distance and cross-section.

   This is known as the radar equation, and is the radar equivalent of the Friis transmission formula.

(b) Calculate the system noise temperature and from that the noise temperature and noise figure of the maser amplifier. You may assume a lossless antenna-preamplifier interface and cable, as well as optimum impedance match.

(c) Explain quantitatively why the subsequent Jet Propulsion laboratory experiment, using a similar antenna and only 13 kW of power at a frequency of 2388 MHz, was successful. What was the maximum usable bandwidth (for S/N=1) in the JPL experiment if they had a receiver of noise figure $N = 0.9dB$?